THE TWENTIETH CENTURY ARITHMETIC.

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PART I.

FOR

PRIMARY SCHOOLS.

BY

ERNEST E. WEST, B.L.,
Graduate U. S. Naval Academy.

ATLANTA, GA.:
THE FOOTE & DAVIES COMPANY,
1897.

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3194-C2,1

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PREFACE TO PART I.

THE idea of number arises when we limit quantity. The child and the savage have very imperfect ideas of number, because they are little concerned with estimating quantity. But as soon as they think of adapting means to an end, a more or less definite idea of measurement begins and thus the idea of number originates. It is evident, then, that to make the child's idea of number definite and clear, the learning of number should begin with measuring some quantity. True ideas of number arise through the activity of the mind in dealing with objects *in a constructive way*.

Just as words symbolize things, so numbers symbolize how many things; and the same method that is used in teaching new words may be used in teaching new numbers. A word is taught by (1) showing the object; (2) speaking the name; (3) writing the name. A number may be taught by (1) counting the objects; (2) speaking the number; (3) writing the number. By persistly following this routine, the successive steps become familiar and proficiency is gradually attained.

It is essential that the comparative values of the first ten numbers be fully appreciated; but a complete analysis and appreciation of the value of large numbers is impossible.

For learning these first ten numbers each pupil should have a hundred cubic centimeter blocks of wood and a rule one decimeter long divided into centimeters. The cost of these (about 25 cents) should not for a moment be thought of when compared to their real value as aides to the learner. The decimeter and cubic centimeter blocks are better than a foot and cubic inch blocks because, the relation of the former is the same as our scale of notation. The teacher should also have an abacus. This apparatus is certainly as much necessary to the teacher of primary arithmetic as proper apparatus is to the teacher of physics.

When the pupil has learned the first ten numbers, the teacher may follow the steps as given in this volume with the assurance that a knowledge of our system of writing numbers and operations with them is being unfolded to the child in its proper order.

ERNEST E. WEST.

ATLANTA, GA., *May 26, 1897.*

CONTENTS OF PART I.

	PAGE.
Arabic System	2
Numeration	5
Review	9
The Roman System	12
Review	15
Diversions	16
Reduction	16
Addition	20
Definitions	27
When the sum of any column is more than 9	28
Review	33
Diversions	34
Multiplication	34
Definitions	40
To Multiply by 10, 100, etc.	43
To Multiply by any number	46
Review	51
Diversions	52
Subtraction	53
Definitions	55
Review	58
Addition and Subtraction	59
Diversions	61
Division	62
Definitions	63
Short Division	64
Long Division	68
To Divide by 10, 100, etc.	70
Averages	72
Review	73
Signs of Aggregation	76
Review of Fundamental Operations	77
Some Properties of Numbers	79
Casting Out Nines	80
Diversions	82
Tests of Divisibility	83
Factors	85
Greatest Common Divisor	88
G. C. D.—Second Method	90
Least Common Multiple	95
L. C. M.—Second Method	98
Review	100

The Twentieth Century Arithmetic.

Part I.

	Question.	Number.		Question.	Number.
/	How many?	**One.**	///// /	How many?	**Six.**
//	How many?	**Two.**	///// //	How many?	**Seven.**
///	How many?	**Three.**	///// ///	How many?	**Eight.**
////	How many?	**Four.**	///// ////	How many?	**Nine.**
/////	How many?	**Five.**	///// /////	How many?	**Ten.**

1. A **number** is a direct answer to the question "How many?" The science of numbers is called **arithmetic.**

Arithmetic explains the simple properties of numbers and how to use them in making calculations.

People could not talk and could even think only a little if they did not have *words;* words are simply *aids* to talking and thinking. So, without *numbers,* people could calculate only a very little; numbers are simply aids to calculating.

2. NOTATION AND NUMERATION.

It would be very inconvenient to express a number always by dots or by marks; therefore certain characters

are used to express a few numbers and, combining these characters in certain ways, we express *any* number.

This is just like expressing certain sounds with characters, called letters, and combining these letters to form words.

Numbers are expressed by characters in two systems—the **Arabic** and the **Roman.**

The Arabic system, because of its superiority, is used in making all calculations. The characters it employs are called **figures.**

The Roman system is used chiefly in numbering chapters, time-pieces, kings, popes, and sometimes dates. The characters it employs are our ordinary capital letters.

Notation is expressing a number with *characters.*

Numeration is expressing a number with *words.*

---

3. ARABIC SYSTEM.

SCALE.—A **unit** is one; as, one apple, one hundred, one x, one mark. *Unus* in Latin means one.

1 Thousand.		1 Hundred.		1 Ten.	1 Unit.
Hundred	*Hundred*	*Ten*	*Ten*		
Hundred	*Hundred*	*Ten*	*Ten*	////	
Hundred	*Hundred*	*Ten*	*Ten*		/
Hundred	*Hundred*	*Ten*	*Ten*	////	
Hundred	*Hundred*	*Ten*	*Ten*		
10 Hundreds.		*10 Tens.*		*10 Units.*	

A group of partridges is called a covey, a group of sticks is called a bundle, a group of cows is called a herd; so in numbers,

A group of 10 units is called a ten.

A group of 10 tens is called a hundred.

A group of 10 hundreds is called a thousand.

We see from this that the **scale** of the Arabic system is **10,** from which fact it is also called the **decimal** system.

The Latin word *decem* means ten.

---

4. NOTATION.—The figures used in this system are

FIGURES.	0	1	2	3	4	5	6	7	8	9
NAMES.	**Naught**	**One**	**Two**	**Three**	**Four**	**Five**	**Six**	**Seven**	**Eight**	**Nine**

Each figure represents one more than the preceding figure. Naming them in their order of increase is called **counting.**

5. All English words are formed by using only twenty-six letters; *all Arabic numbers are formed by using only these ten figures.* Numbers can be expressed by using more or fewer than ten figures; in fact, the system using twelve figures possesses some advantages over that using ten; but the decimal system is the only one used and consequently the only one that need be learned. Just as a word is written by placing its letters in their proper order, so *a number is written by placing its figures in their proper order.*

If we change the order of the letters in *cat* to *act*, we change the word; so, also, if we change the order of figures in a number, we change the number. The orders are numbered in succession from the right. 1492 (one four nine two) is a number in which 2 is in the 1st order, 9 in the 2d. What is in the 3d? the 4th? If we change the order of the figures to 2419, we make quite a different number.

6. *Write the following numbers in figures:*

ORDERS.	**5th.**	**4th.**	**3d.**	**2d.**	**1st.**
1.		one	seven	seven	six
2.			three	six	five
3.		five	two	eight	naught

Orders.	5th.	4th.	3d.	2d.	1st.
4.	two	four	eight	nine	nine
5.	three	one	four	one	six
6.	four	three	five	six	naught
7.	eight	four	two	six	three
8.		one	naught	nine	three
9.	seven	five	one	two	five
10.	nine	four	nine	five	nine
11.		one	four	nine	two
12.	three	one	four	one	six
13.	two	nine	one	three	naught
14.	one	six	naught	nine	three
15.	nine	four	six	two	five
16.	three	two	eight	naught	eight
17.	three	nine	three	seven	naught
18.	two	two	naught	four	six
19.	nine	naught	eight	one	six
20.	nine	one	four	three	nine

All the figures except naught are called significant figures, or **digits.** Naught, called also zero or cipher, means *none*, and is used when no significant figure for an order is given.

7. *Write the following numbers in figures:*

Orders.	4th.	3d.	2d.	1st.
1.		one	five	none
2.		four	none	five
3.	one	six		seven
4.	five	two	eight	
5.	four			four

6. five in 5th order, eight in 4th.
7. two in 3d order, five in 1st.
8. four in 4th order, four in 2d.
9. seven in 5th order, three in 3d.
10. nine in 3d order, one in 1st.

11. two in 3d order.
12. five in 2d order.
13. three in 5th order.
14. six in 4th order.
15. nine in 3d order.
16. nine in 4th order.
17. two in 5th order, nine in 4th, two in 1st.
18. five in 5th order, nine in 4th, three in 3d.
19. five in 6th order, four in 4th, two in 2d.
20. six in 6th order, five in 5th, two in 2d.
21. four in 7th order, two in 4th, one in 1st.

---

8. Numeration.

Each order is given a name for the same reason that all objects are named—so that we may talk and write about them. The 1st order is called **units,** the 2d **tens,** the 3d **hundreds,** the 4th **thousands.**

Orders.	4th	3d	2d	1st.
Number.	1	4	9	2
Names.	**Thousands**	**Hundreds**	**Tens**	**Units**

9. *A number is read by naming its orders,* beginning on the left.

1492 is 1 thousand 4 hundreds 9 tens 2 units.

The name units is always omitted, and usage requires the following irregularities:

Instead of reading 11, one ten one; 12, one ten two; 13, one ten 3, etc., we say

Numbers.	11	12	13	14	15
Names.	**Eleven**	**Twelve**	**Thirteen**	**Fourteen**	**Fifteen**
Numbers.	16	17	18	19	
Names.	**Sixteen**	**Seventeen**	**Eighteen**	**Nineteen**	

Also, instead of reading 20, two tens; 30, three tens; 40, four tens, etc., we say

NUMBERS.	20	30	40	50	60
NAMES.	Twenty	Thirty	Forty	Fifty	Sixty
NUMBERS		70	80	90	
NAMES.		Seventy	Eighty	Ninety	

---

10. EXERCISES.

1. Write all the numbers to twenty, and name them over till you know them perfectly.

2. 21 is read twenty-one; 22 is read twenty-two; 23, twenty-three. Read 24, 25, 26, 27, 28, 29.

3. Write the numbers from 30 to forty and name them.

4. Count from forty to sixty.

5. Name all the tens.

Express in words:			*Write with figures:*	
6. 92	37	27	11. Forty-six.	16. Eighty-four.
7. 65	99	43	12. Sixty-one.	17. Thirty-eight.
8. 76	51	14	13. Seventy-six.	18. Twenty-five.
9. 80	29	25	14. Seventeen.	19. Sixty-six.
10. 16	42	39	15. Ninety-two.	20. Fifty-five.

---

11. HUNDREDS.

It is the general custom to say *and* between hundreds and tens of each period, but the better way is to omit it. In reading decimals (Art. 161), *and* is used for the separatrix.

137 is read, one hundred thirty-seven.

Read:			*Write:*
a.	b.	c.	
1. 492.	225.	781.	Two hundred forty-six.
2. 365.	415.	438.	Four hundred sixty-one.

	a.	b.	c.	
3.	776.	865.	962.	Nine hundred seventy.
4.	280.	607.	784.	Three hundred thirty-nine.
5.	216.	733.	635.	Two hundred fifty.
6.	537.	893.	589.	Five hundred twenty-six.
7.	199.	533.	942.	Eight hundred ninety-one.
8.	451.	542.	864.	Seven hundred seventy-six.
9.	929.	325.	769.	Six hundred six.
10.	742.	243.	847.	One hundred one.

---

12. THOUSANDS.

1492 is read, one thousand four hundred ninety-two.

Read: *Write:*

	a.	b.	
1.	1776.	12632.	Four thousand three hundred seventy-one.
2.	1865.	25491.	Eight thousand seven hundred fifty-four.
3.	1733.	16326.	One thousand one hundred eleven.
4.	5280.	34795.	Five thousand four hundred eighty-two.
5.	2533.	75534.	Two thousand eight hundred thirty-five.
6.	3542.	61863.	Nine thousand two hundred sixty-seven.
7.	4325.	45947.	Six thousand five hundred ninety-three.
8.	6243.	84178.	Three thousand nine hundred nineteen.
9.	9181.	59459.	Twelve thousand three hundred forty-six.
10.	7555.	99682.	Seven thousand six hundred eighteen.

---

13. The following table shows all the orders in practical use. *This is the key to numeration.* Memorize the names of the orders and periods. H. stands for Hundred.

Numeration Table.

Periods.						
Number.	5th.	4th.	3d.	2d.	1st.
Name.	Trillions.	Billions.	Millions.	Thousands.	Units.
Orders.						
Name.	H. trillions, Ten trillions, Trillions	H. billions, Ten billions, Billions	H. millions, Ten millions, Millions	H. thousands, Ten thousands, Thousands	Hundreds, Tens, Units
Number.	15th 14th 13th,	12th 11th 10th,	9th 8th 7th,	6th 5th 4th,	3d 2d 1st.

Each group of three orders, beginning with the 1st order, is called a **period.** The periods of a number are separated from each other by commas. The period on the left may contain only one or two orders. The names of the periods above trillions in their order are quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, etc.

14. *A number is read by naming its orders*, beginning on the left. Usage, however, requires us to say but once for each period that part of the name which is the same.

125,000 is one hundred thousand twenty thousand five thousand, but usage shortens it to one hundred twenty five thousand.

Read:	a.	b.	c.
1.	241,000.	92,000,000.	92,241,000.
2.	325,000.	76,000,000.	76,325,000.
3.	18,000.	42,000,000.	42,018,000.
4.	11,000.	143,000,000.	143,011,000.
5.	183,000.	365,000,000.	365,183,000.

15. Rule for numeration: *Separate the figures into periods. Beginning on the left, read each period.*

62,622,250 is read sixty-two million six hundred twenty-two thousand two hundred fifty.

	Read:		*Write:*		*Read:*
1.	5,280	6.	241 thousand	11.	265,780
2.	31,416	7.	325 thousand	12.	3,141,592
3.	43,560	8.	18 thousand	13.	3,566,480
4.	24,899	9.	11 thousand	14.	62,622,250
5.	84,263	10.	183 thousand	15.	1,479,488,675

16. When 1 is in the 4th order it is usually read with the hundreds. Other figures and orders may be similarly read.

1492 is read fourteen hundred ninety-two.

5280 is read fifty-two hundred eighty.

	Read:		*Write:*
1.	1776	11.	Sixteen hundred sixty-four.
2.	1066	12.	Seventeen hundred twenty-eight.
3.	1865	13.	Fifteen hundred sixty-five.
4.	1607	14.	Nineteen hundred.
5.	1733	15.	Eighteen hundred forty-five.
6.	1893	16.	Seventeen hundred sixty.
7.	2533	17.	Twenty-seven hundred forty.
8.	3542	18.	Fifty-two hundred.
9.	4325	19.	Seventy-five hundred thirty-one.
10.	6243	20.	Ninety-three hundred eighty-two.

---

17. REVIEW.

1. What is a number? What is arithmetic?
2. What are the systems of notation and their uses?

3. Of what use are numbers? What are consecutive numbers?

4. Give another name for the Arabic system. What is its scale?

5. *Name the orders from units to billions.*

6. *Name the periods from trillions to units.*

7. Give the rule for writing a number. How are the orders numbered?

8. Give the rule for numeration.

Read the following sentences:

9. Columbus discovered America in 1492.

10. The first English settlement was made at Jamestown in 1607.

11. Georgia was settled at Savannah in 1733.

12. There are 5280 feet in a mile.

13. The Ferris Wheel at The World's Fair was 265 feet high and could carry 1368 people.

14. The Washington Monument is 555 feet high and the Eiffel Tower at Paris is 975 feet high; the great Watkin Tower in England will be 1150 feet high.

15. It is 24,899 miles around the earth.

16. In an acre there are 43,560 square feet.

17. The moon is about 238,800 miles from us.

18. Light travels 186,500 miles in a second and it takes 493 seconds for the sun's light to reach the earth.

19. The sun is about 92,380,000 miles from the earth.

20. The Bible contains 66 books, 1,189 chapters, 31,173 verses, 773,746 words, and 3,566,480 letters. The word *and* occurs 46,277 times.

21. The population of the U. S. in 1890 was 62,622,250, consisting of 32,067,880 males and 30,554,370 females.

Write the numbers in the following sentences:

22. St. Augustine was founded by the Spaniards in fifteen hundred sixty-five.

23. The area of Georgia is fifty-eight thousand square miles.

24. The population of Chicago in 1890 was one million ninety-nine thousand eight hundred fifty.

25. By the census of 1890, the population of the United States was sixty-two million six hundred twenty-two thousand two hundred fifty.

26. The population of the world in 1890 was estimated to be one billion four hundred seventy-nine million four hundred eighty-eight thousand six hundred seventy-five.

27. In 1894 the Southern States raised nine million thirty five thousand three hundred seventy-nine bales of cotton. The largest crop ever produced.

18. *Read:*

1.	25,004	6.	62,622,250
2.	42,070	7.	92,380,000
3.	806,400	8.	1,479,488,675
4.	984,002	9.	8,701,025,111
5.	101,210	10.	47,842,600,018

19. *Write in figures:*

1. Sixteen thousand five.
2. Eight hundred thousand seventy-three.
3. Three million eleven thousand seventeen.
4. Twenty million fifteen thousand one.
5. One million one hundred one thousand four hundred three.
6. 13 million 147 thousand 218.
7. 62 million 622 thousand 250.
8. 142 million 7 thousand 15.

9. 5 billion 17 million 107 thousand 143.
10. 27 billion 437 thousand 101.
11. 5 thousand 280.
12. 314 thousand 159.
13. 2 million 2 thousand 2.
14. 50 million 50 thousand 50.
15. 250 million 250 thousand 250.
16. Which is the larger number, 41 or 14?
17. Arrange the figures 1, 4, 9, 2 to make the largest number possible. Is it not 9421? Arrange them to make the smallest number possible. Is it not 1249?

Make the largest and smallest possible numbers, using all the figures:

18. 1, 7, 7, 6.
19. 5, 2, 8, 1.
20. 3, 1, 4, 5.
21. 6, 7, 8, 9.
22. 1, 6, 0, 7.
23. 5, 3, 5, 6, 7.
24. 4, 9, 7, 2, 1.
25. 2, 0, 5, 3, 6.

THE ROMAN SYSTEM.

NOTE.—In inserting the explanation of this system here, the author assumes that the pupils know a little addition and subtraction. Should they not, its study must be postponed till after addition and subtraction have been studied.

20. This system employs the following letters:

LETTERS.	I	V	X	L	C	D	M
VALUES.	One	Five	Ten	Fifty	One hundred	Five hundred	One thousand

This is the key to the system: commit it to memory.

These letters may be combined to represent any number There are four principles of combination.

I.—*Repeating a letter repeats its value.*

A letter is not usually repeated more than three times, though 4 on time-pieces is written IIII. V, L, and D are never repeated; VV would mean X, LL would mean C, and DD would mean M.

= means *equals*.

XX=two tens=20. CCC=three hundred=300.

	Read:		*Write:*	
	a.	**b.**	**c.**	**d.**
1.	XX	CCC	two	three thousand
2.	II	XXX	thirty	three hundred
3.	CC	MMM	three	two thousand
4.	MM	III	twenty	two hundred

II.—*When a letter is placed* **after** *one of greater value, their values are added.*

VI=5 and 1=6. LX=50 and 10=60. DC=600.

	Read:				*Write:*	
	a.	**b.**	**c.**	**d.**	**e.**	**f.**
5.	VI	XV	CX	DL	51	105
6.	XI	LV	DX	ML	6	510
7.	LI	CV	MX	CL	15	60
8.	CI	DV	LX	MC	11	55

Combining I. and II. we have:

VII=5 and 2=7. LXXX=50 and 30=80.

	Read:			*Write:*	
	a.	**b.**	**c.**	**d.**	**e.**
9.	XII	LXX	MDCC	12	1700
10.	CII	MXX	MDXX	53	1520
11.	LIII	CXX	MDII	102	1502
12.	DXX	MCC	DCCC	70	800

III.—*When a letter is placed* **before** *one of greater value, its value is subtracted from that of the greater.*

IV=1 from 5=4. XL=10 from 50=40.

	Read:		*Write:*	
	a.	**b.**	**c.**	**d.**
13.	IV	XL	90	400
14.	IX	XC	49	9
15.	IL	CD	4	40

A letter placed *between* two of greater value is taken with the right-hand one according to III.

XIV=X and IV (*not* XI and V)=14.

	Read:		*Write:*	
	a.	**b.**	**c.**	**d.**
16.	XIX	DXL	14	59
17.	LIX	CIX	19	540
18.	LIV	MCD	54	104

IV.—*A bar over a letter, or combination of letters, increases its value a thousand times.*

$\overline{V}$=5 thousand. $\overline{IX}$=9 thousand.

	Read:			*Write:*	
	a.	**b.**	**c.**	**d.**	**e.**
19.	$\overline{X}$	$\overline{IV}$	$\overline{C}$	2 thousand	10 thousand
20.	$\overline{L}$	$\overline{XL}$	$\overline{XV}$	7 thousand	17 thousand

21. The following table is arranged decimally:

	Thousands.	**Hundreds.**	**Tens.**	**Units.**	
1.	M	C	X	I	=1111
2.	MM	CC	XX	II	=2222
3.	MMM	CCC	XXX	III	=3333
4.	$\overline{IV}$	*CD	XL	IV	=4444

*400 is also written CCCC.

5.	$\overline{\text{V}}$	D	L	V	=5555
6.	$\overline{\text{VI}}$	DC	LX	VI	=6666
7.	$\overline{\text{VII}}$	DCC	LXX	VII	=7777
8.	$\overline{\text{VIII}}$	DCCC	LXXX	VIII	=8888
9.	$\overline{\text{IX}}$	CM	XC	IX	=9999

Any number less than ten thousand may be written by using this table. There is no Roman character to represent *none*.

ROMAN. M CD XC IX =MCDXCIX

ARABIC. 1 thousand 4 hundred 9 tens 2 units=1492

	Write in figures:			*Write in letters:*	
	a.	**b.**	**c.**	**d.**	**e.**
10.	MC	DLXII	MCDXCII	1565	1900
11.	DVI	MDC	MDCCLXXVI	1733	1845
12.	$\overline{\text{IV}}$	CXVI	MDCCCLXV	1066	1607
13.	CCL	MCCL	CMXCIX	1815	1492
14.	XCV	MDCC	MDCLXVI	1890	1776

21a. REVIEW.

1. *Give the four principles of combination used in the Roman notation.*

2. Write a table of Roman notation decimally to ten thousand.

Read the following:

3. Pope Leo XIII.
4. King Louis XIV.
5. King Henry VI.
6. Pope Pius IX.
7. Exodus XX. 12.
8. Proverbs VIII. 11.
9. Proverbs XXIII. 12.
10. Psalms XXXII. 8.

11. XLVII. men in 3 years translated the King James I. edition of the Bible. The title page of the first one could have borne the date MDCX.

12. The first book printed in England with a date was printed by William Caxton in the year MCDLXXVII.

13. The inscription on the Liberty Bell was taken from Leviticus xxv. 10.

14. The date on the Liberty Bell is MDCCLIII.

Diversions.

15. Take one from nine and leave ten. IX; X.

16. Add one to nine and make twenty. IX; XX.

17. Take one from nineteen and leave twenty. XIX; XX.

18. Add one to twenty and make nineteen. XX; XIX.

19. Take *nine* from *six*, *ten* from *nine*, and *fifty* from *forty* and *six* will remain.

SIX	IX	XL
IX	X	L
S	I	X

20. To fifty add zero, then five, then a fifth of eight, and the total is the sum of human happiness. LOVE.

---

22. REDUCTION.

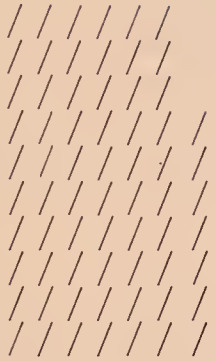
///// ///// ///. Here are thirteen units. Write the number, 13; it represents one ten, three units. 13 *units* is equivalent to 1 *ten*, 3 *units*. This process of changing the name but not the value is called **reduction**.

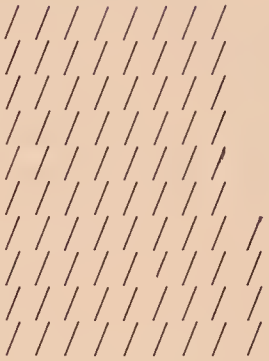
///// ///// ////. How many units? Write the number, 14; read it as units; as tens and units. Read 25 units as tens and units. How many tens in the number 45? How many tens in 130? Are there not 13 tens? How many units in 15 tens? How many tens in 150 units?

1.

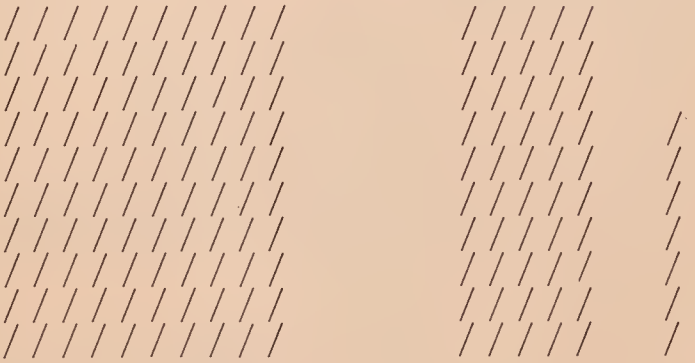
2.

3.

4. 

5. 

Write the number which expresses how many units in each. How many tens are in each? Notice that the left-hand figure in the number of units expresses the number of tens. Also, that 10 units make 1 ten.

6. 

157 units.
15 tens 7 units.
1 hundred 5 tens 7 units.

22a. The first four orders are **Th. H. T. U.** Where there are more than 9 of any order, the left-hand figure is the next higher order.

In 92 units there are 9 tens.
In 49 tens there are 4 hundreds.
In 14 hundreds there is 1 thousand.

1. How many tens in 92? 45? 96? 32? 58?

2. How many hundreds in 49 tens? 21 tens? 43 tens? 75 tens? 18 tens? 25 tens? 67 tens?

3. How many thousands in 14 hundreds? 69 hundreds? 82 hundreds? 43 hundreds? 91 hundreds?

4. How many units in 345? how many tens?

5. How many tens in 1492? how many hundreds?

Fill out the blanks:

	Units.		T.	U.		Tens.		H.	T.	U.
6.	13	is	1	3	11.	75	is	7	5	0
7.	25	is			12.	43	is			
8.	65	is			13.	80	is			
9.	92	is			14.	91	is			
10.	37	is			15.	56	is			

		T.	U.
16.	42 is 3 tens and how many units?	3	12
17.	23 is 1 ten and how many units?		
18.	56 is 4 tens and how many units?		
19.	42 is 3 tens and how many units?		
20.	65 is 5 tens and how many units?		

21. Write the number which has 3 tens, 2 units. *Ans.* 32.

22. Write the number which has 1 ten, 15 units. *Ans.* 25.

Write the number which has

	T.	U.		T.	U.		T.	U.
23.	3	12	26.	5	13	29.	5	10
24.	4	13	27.	8	12	30.	9	10
25.	1	19	28.	1	15	31.	9	15

	H.	T.	U.		H.	T.	U.		H.	T.	U.
32.	3	1	15	35.	3	12	5	38.	4	11	13
33.	5	3	12	36.	4	17	0	39.	6	13	19
34.	4	8	12	37.	5	10	0	40.	9	9	10

	Th.	H.	T.	U.		Th.	H.	T.	U.
41.	1	4	8	12	51.		1	32	0
42.	1	6	16	16	52.	2	37	0	0
43.	5	2	7	10	53.	3	22	7	10
44.	1	5	9	17	54.		51	18	0
45.	3	21	6	24	55.		16	16	16
46.		14	25	13	56.		9	9	10
47.	3	21	6	24	57.	5	9	10	0
48.	5	10	2	10	58.		12	29	2
49.	1	3	18	12	59.	3	25	10	17
50.			6	25	60.	4	31	42	10

Combine (add) the following, writing as one number:
400, 2, 1000, and 90. *Ans.* 1492.

61. 20 and 5.
62. 7 and 40.
63. 80 and 1.
64. 9 and 30.
65. 4 and 70.
66. 300, 60, and 5.
67. 200, 5000, and 80.
68. 1700, 70, and 6.
69. 92, 400, and 1000.
70. 700, 28, and 1000.
71. 170000, 1, and 3900.
72. 100000, 500, and 86000.
73. 3600000, 160, 7, and 68000.
74. 62000000, 250, 600000, and 22000.
75. 2300000, 90000000, and 80000.

---

23. Scale of the Arabic Notation.

A unit is one; as, one apple, one dollar, one.

Orders:	5th.	4th.	3d.	2d.	1st.
	2	7	5	4	3

In the number 27543 there are three units of the first order; 4 units of the second order; how many units of the third order? of the 4th? of the 5th?

The largest number of units of any order that can be expressed by a single figure is nine. How, then, can we express more than 9 units of any order? The answer is, we cannot express it with one figure, so we always reduce it to units of the next higher order.

Ten units of any order make one unit of the next higher order.

This principal is used in adding, subtracting, multiplying, and dividing numbers.

---

24. ADDITION.

/////. Here are 5 marks, make 3 more and count as you make them: ///// / 6; ///// // 7; ///// /// 8.

You have found out by counting that the sum of 5 and 3 is 8. This is the only way to find the sum of two *digits*, but by remembering the sums of every pair of digits, there is a process called **addition** by which the sum of *any* two numbers may be found.

The sign of addition, +, is called **plus.**

$5 + 3 = 8$ is read 5 plus 3 equals 8.

1. If you make 5 marks and then 4 more, how many will there be?

2. John put in his bank 7 cents at one time and 8 cents at another; how many cents did he put in?

3. Roy rode 9 miles on his bicycle in the morning and 8 miles in the afternoon; how far did he ride during the day?

4. How many are 8 apples and 6 apples?

5. How many are $5x$ and $7x$? Make 5 x's, then 7 more, and count how many in all.

Adding is performed oftener than any other operation in arithmetic; it should, therefore, be very thoroughly learned. The name of a *familiar* object, a book, for instance, comes to the mind at once, without effort, on seeing it; so ought the sum of every pair of digits on seeing them. Much practice, *becoming familiar with them*, is the only way of obtaining this result.

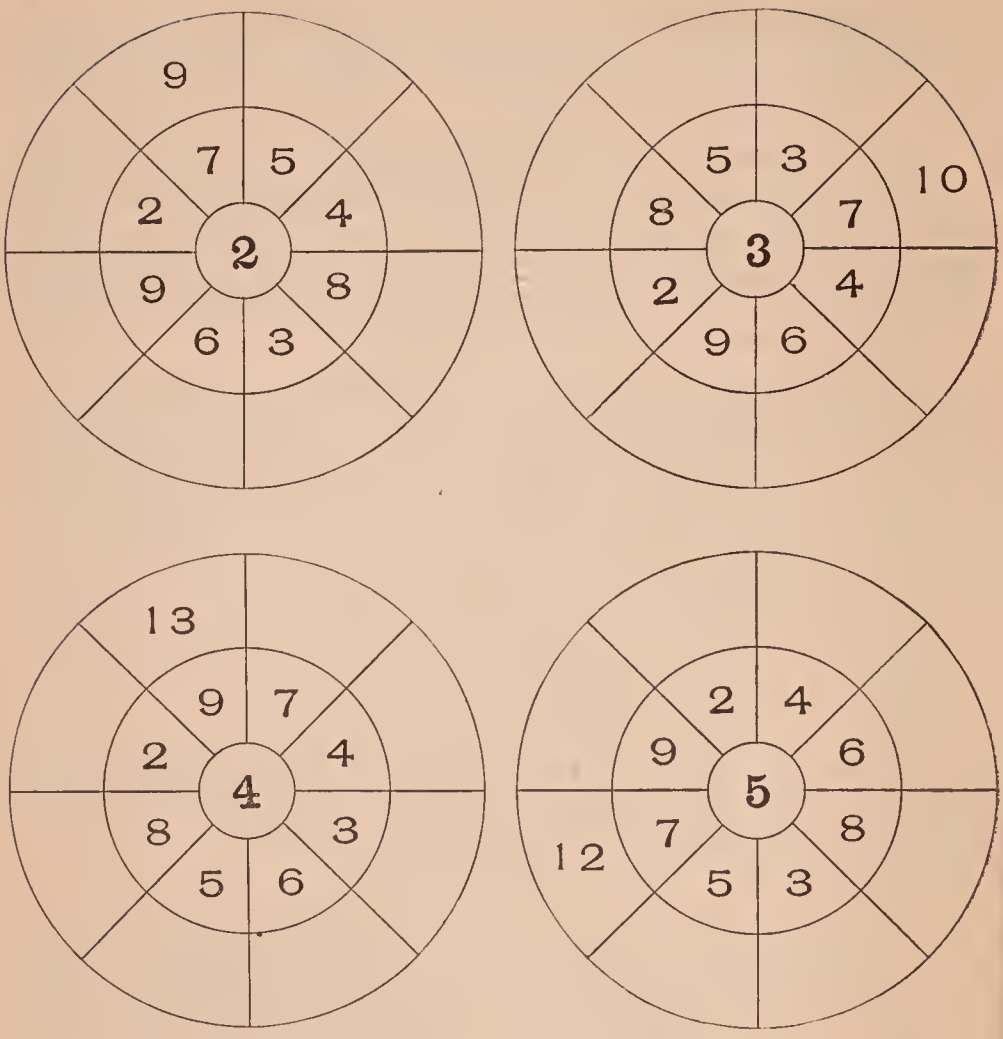
Memorizing the sums of every pair of digits is a very great task. Memory is strengthened by *attention* and by *repetition*. Attention can be obtained by making the subject attractive and this will depend on the genius of the teacher. Pupils should not be required to memorize many sums each day and they should be required to find the sums by counting. They will then appreciate the value of the number and also the labor saved by memorizing the sums. Let pupils make diagrams similar to those below and fill in the blank spaces. Attention should be constantly called to the commutative law of addition; example, if $9 + 5 = 14$, then also $5 + 9 = 14$.

Perfection in addition must be attained sooner or later, but this perfection must not be striven for by excluding all other ideas and operations. The repetition of addition in other operations will eventually fix it indelibly on the mind of the pupil.

	7	2	5	3
3				
5		7		
2				5
6				

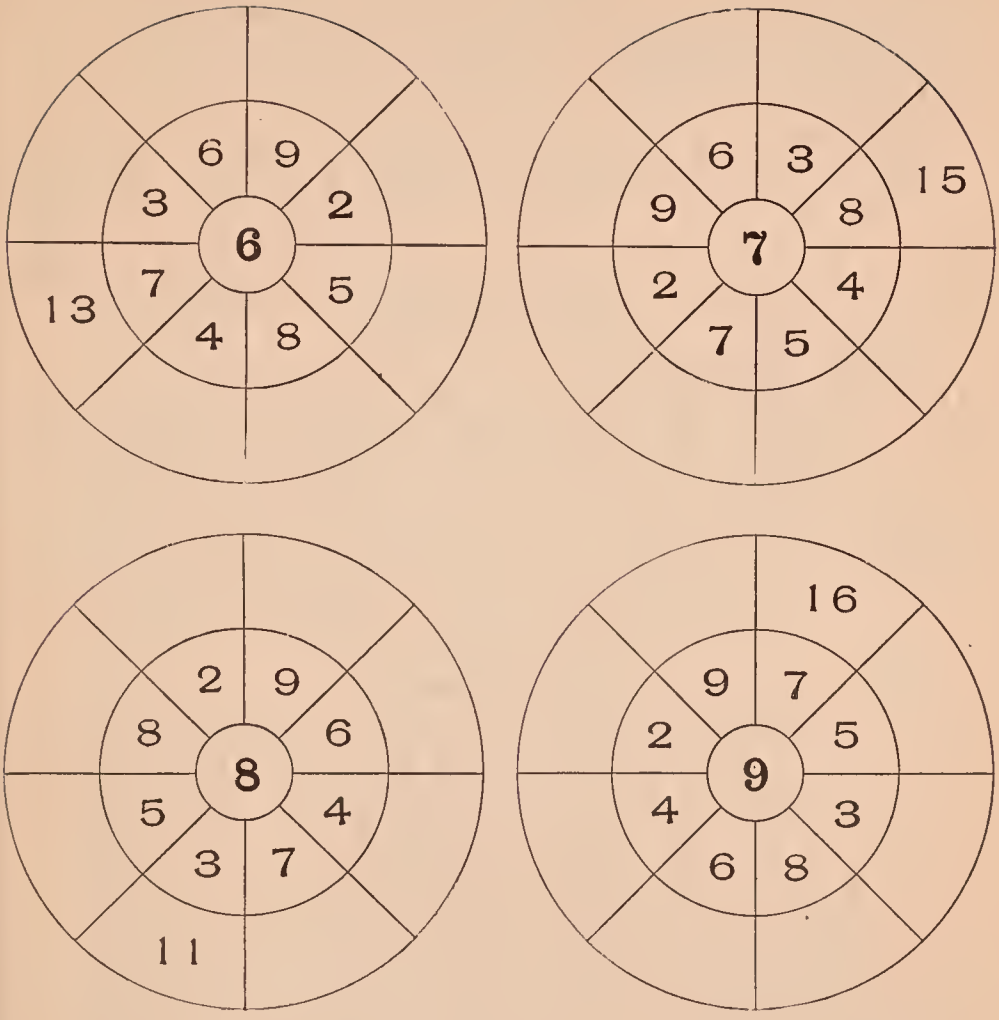
	7	4	8	6	9
7		11			
8					17
4			12		
9					

Fill out the blanks.



The Arabic method of notation is generally believed now to have been invented by the Hindoos.

The nine digits whose origin has been the subject of much research have been attributed by some able scholars to the initial letters of the Sanscrit numerals in analogy to the Grecian and Roman method of representing numbers by initial letters. This is the most recent and probable theory.



The great Italian poet, Petrarch, has left us the oldest authentic date written in numeral characters: the date is 1375, written upon a copy of St. Augustine. The earliest inscription on a monument in these characters in England is in the Church of Ware on a brass plate commemorating the death of Ellen Wood 1454

Euclid was the first writer on arithmetic whose writings have come down to us.

Pythagoras lived about 600 B. C. He is the reputed inventor of the multiplication table.

The following combinations are the foundation of all arithmetical operations.

The sum less than 10:

1	2	3	2	4	3	5	4	3	6
1	1	1	2	1	2	1	2	3	1
—.	—.	—.	—.	—.	—.	—.	—.	—.	—.
5	4	7	6	5	4	8	7	6	5
2	3	1	2	3	4	1	2	3	4
—.	—.	—.	—.	—.	—.	—.	—.	—.	—.

The sum 10 or more:

9	8	7	6	5	9	8	7	6	9	8	7
1	2	3	4	5	2	3	4	5	3	4	5
—.	—.	—.	—.	—.	—.	—.	—.	—.	—.	—.	—.
6	9	8	7	9	8	7	9	8	9	8	9
6	4	5	6	5	6	7	6	7	7	8	8
—.	—.	—.	—.	—.	—.	—.	—.	—.	—.	—.	—.

---

25. **Addition Table.**

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

This table is for reference from the exercises below. The teacher should explain how to use the table.

Point between the following digits and announce the sum of the two digits between which the pointer is.

6, 2, 2, 3, 9, 5, 6, 6, 3, 8, 8, 2, 5, 7, 6, 9, 9, 2, 7.

5, 8, 9, 7, 7, 3, 5, 5, 4, 8, 7, 4, 4, 6, 8, 2, 4, 3, 3.

26.

1. Add 4 to 3, to 13, to 23, to 33, to 43, to 53.
2. Add 5 to 7, to 17, to 27, to 37, to 47, to 57.
3. Add 6 to 5, to 15, to 25, to 35, to 45, to 55.
4. Add 7 to 2, to 12, to 22, to 32, to 42, to 52.
5. Add 8 to 4, to 14, to 24, to 34, to 44, to 54.
6. Add 9 to 6, to 16, to 26, to 36, to 46, to 56.

Add by 2's from 2 to 16, thus: 2, 4, 6, 8, 10, 12, 14, 16.

7. Add by 2's from 1 to 21, thus: 1, 3, 5, 7, etc.
8. Add by 3's from 1 to 31. From 2 to 32.
9. Add by 3's from 3 to 33. From 4 to 34.
10. Add by 4's from 1 to 41. From 3 to 43.
11. Add by 4's from 2 to 42. From 4 to 44.
12. Add by 5's from 1 to 51. From 4 to 54.
13. Add by 5's from 3 to 63. From 5 to 55.
14. Add by 6's from 1 to 61. From 4 to 64.
15. Add by 6's from 2 to 62. From 5 to 65.
16. Add by 6's from 3 to 63. From 6 to 66.
17. Add by 7's from 1 to 71. From 4 to 74.
18. Add by 7's from 2 to 72. From 5 to 75.
19. Add by 7's from 3 to 73. From 6 to 76.
20. Add by 8's from 1 to 81. From 2 to 82.
21. Add by 8's from 3 to 83. From 4 to 84.
22. Add by 8's from 5 to 85. From 7 to 87.
23. Add by 9's from 1 to 91. From 6 to 96.
24. Add by 9's from 2 to 92. From 7 to 97.
25. Add by 9's from 5 to 95. From 9 to 99.

27. I.—Add each of the following columns twice, beginning first at the bottom then at the top. Compare the two sums found for each column.

In adding, name results only. Thus, in example **1,** say 7, 9, 14, 17, **19;** do *not* say 7 and 2 are 9, and 9 and 5 are 14, etc.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
2	4	6	8	3	5	7	9	4	5	6	7
3	7	8	9	2	1	4	3	5	4	6	2
5	1	3	2	7	6	5	9	8	6	2	5
2	9	8	7	3	4	9	2	6	5	3	4
7	4	2	6	2	4	3	7	6	5	3	4

13. $4+8+3+7+2=$
14. $5+9+4+8+3=$
15. $6+1+5+9+4=$
16. $7+2+6+1+5=$
17. $3+2+7+9+3+8=$
18. $5+4+5+8+5+6=$
19. $7+8+3+6+7+4=$
20. $9+6+1+4+9+2=$

II.—Add the following columns first up, then down; and the rows first from the left, then from the right.

	27.	28.	29.	30.	31.
21.	4	4	4	4	4
22.	7	2	3	5	9
23.	3	8	9	2	6
24.	5	5	5	5	5
25.	8	7	5	9	4
26.	4	6	3	6	7

	38.	39.	40.	41.	42.
32.	9	4	3	7	6
33.	7	7	7	7	7
34.	3	5	7	9	6
35.	2	3	4	5	6
36.	6	6	6	6	6
37.	4	5	2	3	6

43.	2	5	6	9	4	2	3	7	6	5
44.	4	7	6	1	8	5	3	0	4	2
45.	7	8	0	3	1	2	4	8	6	5

	56.	57.	58.	59.	60.	61.	62.	63.	64.	65.
46.	6	7	3	1	8	9	7	6	4	9
47.	5	1	7	4	2	7	8	1	8	2
48.	3	8	6	8	5	3	1	9	2	3
49.	0	3	7	9	9	6	4	0	0	3
50.	4	2	4	6	2	0	7	5	9	4
51.	2	7	7	8	5	5	1	8	6	8
52.	9	0	2	5	9	4	7	2	9	7
53.	8	9	9	8	6	0	3	9	3	0
54.	6	4	2	8	3	9	7	2	1	4
55.	3	2	5	1	6	4	7	9	2	8

Frequent practice on these exercises will give the ability to add rapidly and accurately.

DEFINITIONS.

Addition is the process of finding *one* number which contains as many units as two or more numbers.

The numbers added are called **addends.**

The result of addition is called the **sum** or **amount.**

Only like numbers can be added.

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28. ADDING NUMBERS LARGER THAN 9.

I.—When the sum of each column is less than 10.

A farmer sold some corn for 150 dollars, wheat for 223 dollars and cotton for 415 dollars. How many dollars did he receive?

Hund's.	Tens.	Units.
$ 1	5	0
2	2	3
4	1	5
$ 7	8	8

The dollar sign, $, is placed to the left of the number. It should be used with the first addend only.

DIRECTIONS.—Write the numbers so that the same orders shall be under each other. Begin on the right and add each column.

1.	2.	3.	4.	5.	6.
$217	141 a	408	$411	206 x	365
431	218 a	270	263	340 x	402
250	300 a	311	112	231 x	221

7. A farmer's land is worth $2180, his house $1200, and his stock $400. What is it all worth?

8. Three bales of cotton weigh respectively 424, 453, and 510 pounds. How many pounds in all?

9. If Santa Claus has 320 dolls, 303 drums, and 314 books, how many presents has he in all?

10. How many parents does every boy have? In a school there are 213 boys and 211 girls, none of whom are brothers or sisters. How many parents altogether have these children?

11. Solomon's temple was built 1014 years before Christ. If it were still standing how old would it be 1900 years after Christ?

12.	13.	14.	15.	16.	17.
1492	1243	1212	1776	3146	3116
5200	6421	3010	5112	3210	4532
3106	1032	3146	4001	2112	2121

II.—When the sum of any column is more than 9.

Ten units of any order make one unit of the next higher order.

		Th.	H.	T.	U.
13 units	=			1	3
25 tens	=		2	5	
14 hundreds	=	1	4		

Therefore, from the nature of the decimal system, when the sum of any column is more than 9, the left figure of the partial sum is units of the next higher order.

Find the sum of 385, 482, and 796.

21	Directions.—Write the numbers so that the same orders will fall under each other. Draw a line above and below. Begin on the right and add each column. If the sum of any column (except the extreme left one) is more than 9, place the *units* of the sum *under* the column and the *tens above* the next column to the left. Add the number above the column with the column over which it is placed. The work on the right is to explain that on the left.	385
385		482
482		796
796		13 } Partial
1663		25 } sums.
		14
		1663

To detect mistakes in addition, it is well to add each column twice: from the bottom up, and from the top down.

29. 1. In an example in addition, the sum of the units column is 52, of the tens, 24, of the hundreds, 12. What is the total sum? (See Art. 22.)

Find the total sum in each of the following:

	Sum of hundreds.	Sum of tens.	Sum of units.		Sum of hundreds.	Sum of tens.	Sum of units.
2.	36	34	14.	6.	17	23	18.
3.	30	25	5.	7.	21	19	10.
4.	21	12	3.	8.	10	16	24.
5.	15	27	11.	9.	16	21	31.

In writing numbers, care should be taken to make the figures clear and plain, so that 3 will not be mistaken for 5, or 7 for 9. See, also, that all the figures in a column of like orders are placed directly under each other. Always set down the figures to be carried; then, if necessary, any column may be added without the trouble of adding the preceding column.

The ability to add accurately and rapidly is more than worth all the pains necessary to acquire it.

10. Feet.	11. Dollars.	12. Bushels.	13. Pounds.	14. Miles.
1728	449	684	3845	7284
365	876	365	1309	9782
5280	984	492	1849	8447

15. Four bales of cotton weigh respectively 485, 510, 463, and 513 pounds. Find the sum of their weights.

16. The land surface of the globe is composed of the eastern continent, 35,413,799 square miles; the western continent, 16,237,535 square miles; Australia and the islands, 3,709,781 square miles. What is the entire land area?

17. Find the sum of the lengths of the following rivers: Missouri, 2908; Mississippi, 2616; Arkansas, 2170; Red, 2100; Nebraska, 1200; Yellowstone, 1000; and Ohio, 975 miles. (These are the Mississippi and its longest branches.)

18. In one mile there are 5280 square feet; how many feet in five miles?

19. *Find the area of the United States from the following data:*

Area ceded by Great Britain,	1783	830,000
Louisiana purchase from France,	1803	1,182,752
Florida purchase from Spain,	1819	59,268
Annexation of Texas,	1845	274,356
Treaty with Mexico,	1848	522,568
Texan claims from Mexico,	1850	96,707
Gadsden purchase from Mexico,	1853	45,527
Alaska purchase from Russia,	1867	577,390
	Total Area,	

	20. Estimated population of the world in 1897.	21. Number dollars paid by U. S. in pensions 1890–5.
Asia,	825,924,000	106,493,890
Europe,	357,379,000	118,548,859
Africa,	163,933,000	141,086,948
America,	121,713,000	158,155,342
Oceanica,	7,500,400	140,772,163
Australia,	3,230,000	140,959,361
Total,		

	22. Population of cities in 1890, 1891, or 1892.	23. Number bales cotton raised in U. S. 1886–95.
London,	4,231,431	6,550,215
Paris,	2,447,957	6,513,624
New York,	1,801,739	7,017,707
Berlin,	1,579,244	6,935,082
Tokio,	1,389,684	7,313,726
Vienna,	1,364,548	8,655,518
Philadelphia,	1,142,653	9,038,707
Chicago,	1,099,850	6,717,142
St. Petersb'g,	1,035,439	7,527,211
Brooklyn,	957,163	9,892,766
Sum,		

*** Population of United States in 1890.**

States and Territories.	24. Total Population.	25. Sex. Male.	26. Sex. Female.
Alabama,	1,513,017	757,456	755,561
Arizona,	59,620	36,571	23,049
Arkansas,	1,128,179	585,755	542,424
California,	1,208,130	700,059	508,071
Colorado,	419,198	245,247	166,951
Connecticut,	746,258	369,538	376,720
Delaware,	168,493	85,573	82,920
District of Columbia,	230,392	109,584	120,808
Florida,	391,422	201,947	189,475
Georgia,	1,837,353	919,925	917,428
Idaho,	84,385	51,290	33,095
Illinois,	3,826,351	1,972,308	1,854,043
Indiana,	2,192,404	1,118,347	1,074,057
Iowa,	1,911,896	994,453	917,443

*The sum of males and females give the total population. Use this fact to make additional examples.

States and Territories.	27. Total Population.	28. Sex. Male.	29. Sex. Female.
Kansas,	1,427,096	752,112	674,984
Kentucky,	1,858,635	942,758	915,877
Louisiana,	1,118,587	559,350	559,237
Maine,	661,086	332,590	328,496
Maryland,	1,042,390	515,691	526,699
Massachusetts,	2,238,943	1,087,709	1,151,234
Michigan,	2,093,889	1,091,780	1,002,109
Minnesota,	1,301,826	695,321	606,505
Mississippi,	1,289,600	649,687	639,913
Missouri,	2,679,184	1,385,238	1,293,946
Montana,	132,159	87,882	44,277
Nebraska,	1,058,910	572,824	486,086
Nevada,	45,761	29,214	16,547
New Hampshire,	376,530	186,566	189,964
	30.	31.	32.
New Jersey,	1,444,933	720,819	724,114
New Mexico,	153,593	83,055	70,538
New York,	5,997,853	2,976,893	3,020,960
North Carolina,	1,617,947	799,149	818,798
North Dakota,	182,719	101,590	81,129
Ohio,	3,672,316	1,855,736	1,816,580
Oklahoma,	61,834	34,733	27,101
Oregon,	313,767	181,840	131,927
Pennsylvania,	5,258,014	2,666,331	2,591,683
Rhode Island,	345,506	168,025	177,481
South Carolina,	1,151,149	572,337	578,812
South Dakota,	328,808	180,250	148,558
Tennessee,	1,767,518	891,585	875,933
Texas,	2,235,523	1,172,553	1,062,970
Utah,	207,905	110,463	97,442

States and Territories.	33. Total Population.	34. Sex. Male.	35. Sex. Female.
Vermont,	332,422	169,327	163,095
Virginia,	1,655,980	824,278	831,702
Washington,	349,390	217,562	131,828
West Virginia,	762,794	390,285	372,509
Wisconsin,	1,686,880	874,951	811,929
Wyoming,	60,605	39,343	21,362
Total,			

30. REVIEW.

1. What is addition?
2. Name the parts in addition: $5 + 3 = 8$.
3. What is the sign of addition and its name?
4. When the addends are 3, 5 and 8, what is the sum?
5. Write 1492 five times and find the sum.
6. A, B, and C began business together. A put in \$17,500; B \$2,550 more than A; and C as much as A and B together. What was the whole amount put in?
7. How many times must 1760 be used as an addend to give 5280?
8. Find the sum of the five largest numbers that can be expressed by the figures 8, 7, 6, 3, and 5.
9. Find the sum of the five smallest numbers that can be expressed by the figures 1, 2, 3, 7, and 8.
10. A has \$776; B has twice that much and C three times that much; how much have they all together?
11. Use 142,857 as an addend 4 times and find the sum.
12. Use 142,857 as an addend 5 times and find the sum.
13. Use 142,857 as an addend 6 times and find the sum.
14. In one year there are 365 days; how many days in 5 years?

15. In one mile there are 1760 yards; how many yards are in 5 miles?

DIVERSIONS.—Add the rows, columns, and two long diagonals in the following figures and see if the sums for each row, etc., are not the same.

16.

4	9	2
3	5	7
8	1	6

17.

1	16	11	6
13	4	7	10
8	9	14	3
12	5	2	15

18.

3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

31. MULTIPLICATION.

/// /// /// /// ///. Here are 5 groups of marks with 3 marks in each group. Add by 3's and find how many marks in all. You have found by adding that 5 times 3 is 15. This is the way to find the product of two digits. By remembering the product of every pair of digits, there is a process called **multiplication** by which the product of *any* two numbers may be found.

The sign of multiplication is $\times$.

1. 5×3 is read 5 times 3.

How many are

2. // //? Two times 2?
3. // // //? Three times 2?
4. // // // //? 4×2?
5. // // // // //? 5×2?
6. // // // // // //?
7. // // // // // // //?
8. // // // // // // // //?
9. // // // // // // // // //?

10.	a.	b.	c.	d.	e.	f.	g.	h.
	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9

Add these columns and try to remember the sums.

11. How much is 2 taken 3 times? What is 3 times 7? 3 times 4? 3 times 5? 3 times 9? 3 times 6?

If you do not remember these products, add as in Example 10.

12.	a.	b.	c.	d.	e.	f.	g.	h.
	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9

Add these columns and try to remember the sums.

13. How much is 2 taken 4 times? What is 4 times 3? 4 times 7? 4 times 5? 4 times 9? 4 times 4?

Let the pupil construct tables similar to these and thus learn the other multiplication tables. Learning them is a simple act of memory and they should be repeated, little at a time, till memorized well enough to proceed. The repetitions of later work will fix them indelibly in the mind.

14. *What is the difference between*

3 groups of 4 marks and 4 groups of 3 marks?

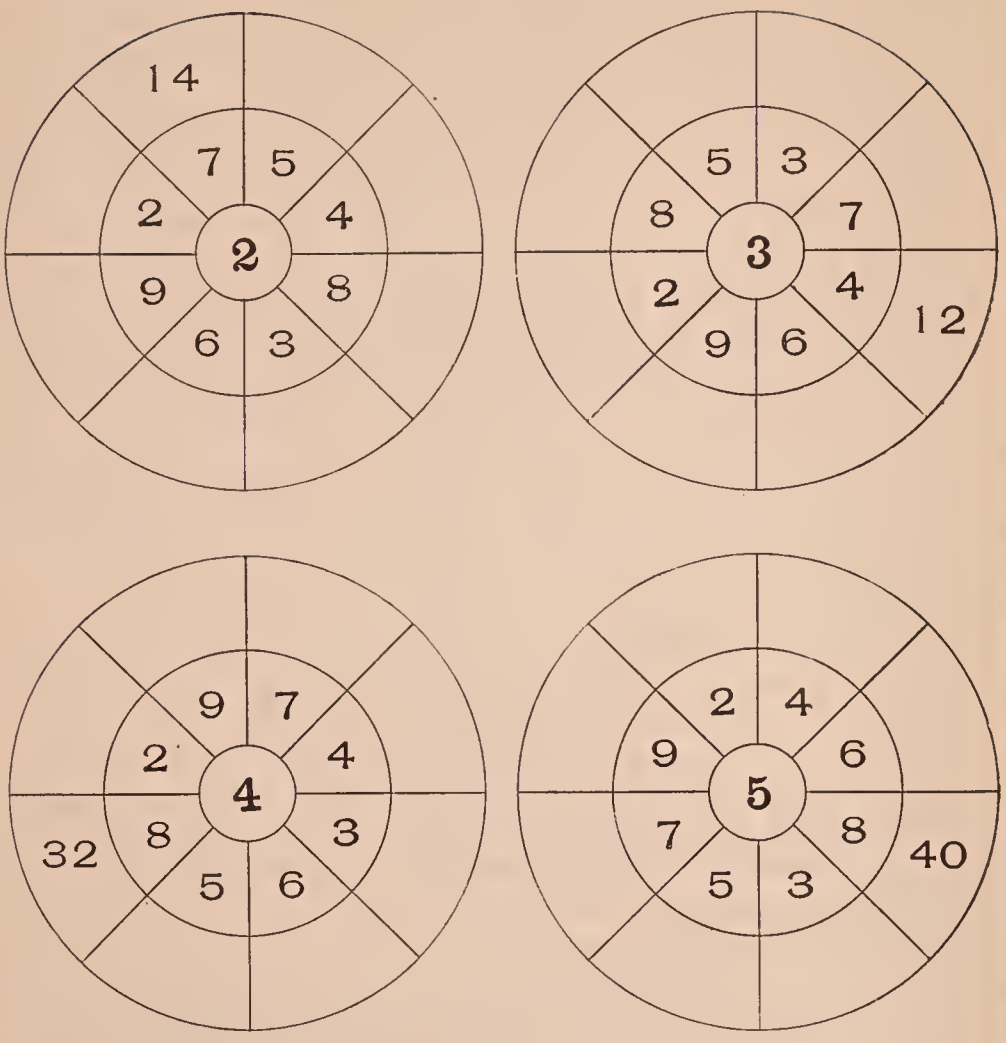
//// //// //// and /// /// /// ///?

3 times 4 · and 4 times 3?

This law of commutation should be noticed for the product of every pair of digits. The labor of learning the multiplication table is thus lessened almost one half.

The product of other digits by 9 are usually the hardest to remember.

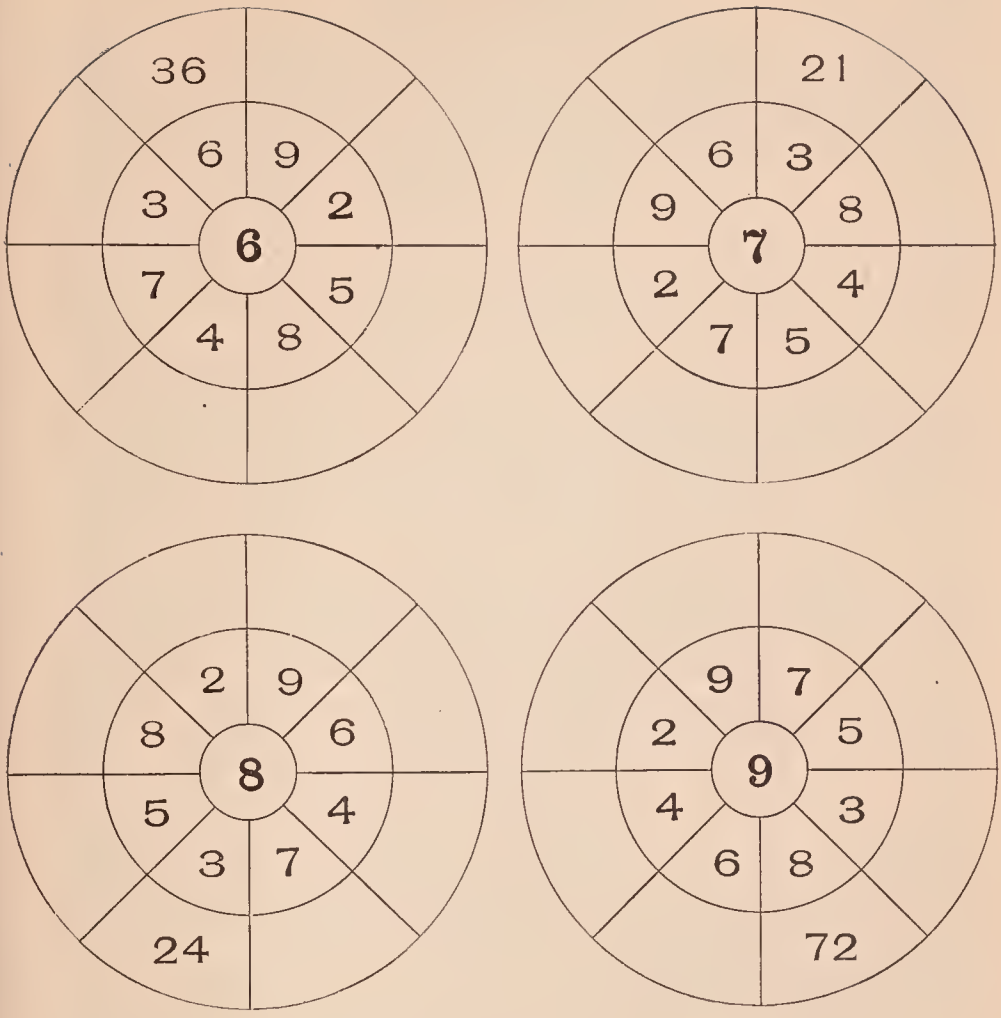
Look at the table below and notice that up to 9×10 the sum of the digits of the products by 9 is always 9, and that the *tens* of the products is always one less than the number multiplied.



The oldest text-book on arithmetic which employs Arabic figures and decimal notation is undoubtedly that of Aricenna, an Arabian physician who lived about 1000 A. D.

The first arithmetic published was in 1478. In 1544, Michael Stifel published his "Arithmetic Integra." He is the acknowledged inventor of the signs $+$, $-$, and $\sqrt{\ }$.

In 1557, Robert Recorde invented and used the sign $=$ in an algebra.



In 1631, Wm. Oughtred in his "Clavis Mathematicæ" first used the sign $\times$.

Dr. John Pell (1610–1658), professor of mathematics and philosophy at Breda, invented $\div$.

Descartes invented exponents. ab $(= a \times b)$ is said to be due to Thomas Harriot, 1631.

Newton invented the binomial theorem.

32. **Multiplication Table.**

	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

This table is for reference.

Nothing added to itself is still nothing.

2×0 is 0.
0×2 is 0.
7×0 is 0.

1. 0×4 is what?
2. 5×0 is what?
3. 7×0 is what?
4. $8 \times 0 = ?$
5. $0 \times 9 = ?$
6. $0 \times 6 = ?$

---

33. PROBLEMS.

1. There are 2 pints in 1 quart; how many pints are there in 7 quarts?

2. If an apple weighs 5 ounces, how much will 9 apples weigh?

3. There are 3 feet in 1 yard; how many feet in 5 yards?

4. A book-case has 7 shelves and there are 9 books on each shelf; how many books in the case?

5. Roy rode 8 miles an hour for 6 hours; how far did he go?

6. There are 6 rows of desks and 8 desks in each row; how many desks in all?

7. The States in the United States would make 9 rows and have 5 States in each row; how many States are there?

8. There are 4 quarts in 1 gallon; how many quarts in 7 gallons?

9. 8 silver dollars side by side extend a foot. On a square foot could be placed 8 of these rows; how many silver dollars could be placed on a square foot?

10. The number of presidents to 1900 could be put in 9 groups of 3 each; how many presidents?

11. How many days in 8 weeks?

12. There are 4 pecks in 1 bushel; how many pecks in 7 bushels?

13. There are 4×7 canals in the United States; how many are there?

14. The Nicaragua canal will require 9×3 miles of excavation.

15. *Point between each pair of the following digits and give the products of each.*

6, 2, 2, 3, 9, 5, 6, 6, 3, 8, 8, 2, 5, 7, 6, 9, 9, 2, 7,
5, 8, 9, 7, 7, 3, 5, 5, 4, 8, 7, 4, 4, 6, 8, 2, 4, 3, 3.

Learn the multiplication table thoroughly. Practice this exercise often.

DEFINITIONS.

Multiplication is a short process for finding the sum when the same number is repeated.

The **multiplicand** is the number repeated.

The **multiplier** is the number which shows how many times the multiplicand is repeated.

The result is called the **product.**

A **concrete number** is a number applied to some *thing.* 3 *marks,* 4 *apples,* 7 *horses,* 5 *x.*

An **abstract number** is simply a number of units. 6, 9, 5.

---

34. $7x + 7x + 7x = 21x$, $7x \times 3 = 21x$. $7 \times 3 = 21$.

Only *like* numbers can be added: the sum of concrete numbers is concrete; the sum of abstract numbers is abstract. Therefore, the multiplicand and product must be either both concrete or both abstract. The multiplier, however, is always abstract, because it only shows how many times the multiplicand is used as an addend.

35. When the multiplier comes first, the multiplication sign is read *times;* when last, the sign is read *multiplied by.*

$3 \times 7x$ is read 3 *times* $7x$. $7x \times 3$ is read $7x$ *multiplied by* 3.

When both numbers are abstract, either may be considered the multiplier.

Name the parts of $3 \times 7x = 21x$. *Ans.* 3 is the multiplier, $7x$ the multiplicand, and $21x$ the product.

36. Multiply 231 by 3.

H.	T.	U.
2	3	1
		3
6	9	3

DIRECTIONS.—Place the multiplier under the units of the multiplicand. Multiply each digit of the multiplicand by the multiplier. 3 times 1 unit is 3 units; 3 times 3 tens is 9 tens; 3 times 2 hundreds is 6 hundreds.

H.	T.	U.
2	3	1
2	3	1
2	3	1
6	9	3

1.	2.	3.	4.	5.
341	212	323	423	211
2	4	3	2	4

Multiply

6. 3122 by 3.	11. 3123 by 3.	16. 3013 by 3.
7. 4324 by 2.	12. 2211 by 4.	17. 2423 by 2.
8. 1312 by 3.	13. 1111 by 8.	18. 1022 by 4.
9. 2121 by 4.	14. 3124 by 2.	19. 3141 by 2.
10. 3223 by 3.	15. 2112 by 4.	20. 1021 by 5.

Th.	H.	T.	U.
5	2	0	7
			3
1	5	6	2	1

EXPLANATION.—3 times 7 units is 21 units, which is 2 tens 1 unit, (Art. 22.) 3 times 5 thousands is 15 thousands, which is 5 thousand and 1 ten thousand.

21. 5207 by 4.	26. 7080 by 9.	31. 4108 by 7.
22. 6109 by 7.	27. 5070 by 6.	32. 8091 by 9.
23. 3208 by 4.	28. 6009 by 8.	33. 7108 by 8.
24. 7091 by 8.	29. 4007 by 7.	34. 9009 by 9.
25. 3081 by 9.	30. 6071 by 9.	35. 8108 by 8.

Multiply 1492 by 9.

1st step.	*2d step.*	*3d step.*	*4th step.*
1492	1492	1492	1492
9	9	9	9
18	18	3618	3618
	81	81	981
			Ans. 13428

EXPLANATION.—*1st step:* 9 times 2 *units* is 1 *ten*, 8 *units*. *2d step:* 9 times 9 *tens* is 8 *hundreds* 1 *ten;* place the tens under each other. *3d step:* 9 times 4 *hundreds* is 3 *thousands*, 6 *hundreds;* place as shown. *4th step:* 9 times 1 *thousand* is 9 *thousands;* place as shown. Add.

Place like orders in the same column.

Multiply

36. 1492 by 8.	**41.** 6485 by 6.	**46.** 7942 by 6.
37. 1492 by 7.	**42.** 7349 by 7.	**47.** 6893 by 4.
38. 1776 by 9.	**43.** 6884 by 8.	**48.** 7265 by 7.
39. 1776 by 8.	**44.** 9447 by 9	**49.** 8496 by 7.
40. 1776 by 7.	**45.** 6553 by 5.	**50.** 9876 by 9.

51. 142,857 by 8.
52. 323,233 by 9.
53. 425,846 by 8.
54. 314,169 by 9.
55. 391,763 by 8.
56. 142,857 by 7.

---

37. Multiply 1728 by 3. Read Art. 22.

1728
3
5184

DIRECTIONS.—Place the multiplier under units of the multiplicand. Multiply each digit of the multiplicand by the multiplier. 3 time 8 units is 24 units, which is 2 tens 4 units. Write down the 4 units and add the 2 tens to the product of tens, etc.

1728
1728
1728
5184

1.	2.	3.	4.
5280	6141	4086	62107
3	7	9	8

Multiply

5. 1093 by 4.	**13.** 31,416 by 8.	**21.** 31,416 by 7.
6. 365 by 7.	**14.** 63,360 by 6.	**22.** 63,360 by 8.
7. 640 by 9.	**15.** 39,370 by 9.	**23.** 39,370 by 6.
8. 5280 by 6.	**16.** 21,504 by 7.	**24.** 21,504 by 9.
9. 6141 by 8.	**17.** 43,560 by 5.	**25.** 43,560 by 7.
10. 4086 by 5.	**18.** 48,667 by 4.	**26.** 48,667 by 9.
11. 7925 by 7.	**19.** 46,656 by 7.	**27.** 46,656 by 8.
12. 2902 by 7.	**20.** 36,524 by 4.	**28.** 36,524 by 7.

29. In every yard there are 3 feet; how many feet are in 1760 yards?

30. Multiply 142,857 by 2, by 3, and by 4. Notice the curious fact that the products consist of the same figures in the same order, beginning at a different place.

31. Multiply 142,857 by 8. In the product, remove the left hand digit and add it to the right hand one; notice the curious result.

32. When cotton is worth 8 cents a pound, how much is a bale weighing 475 pounds worth?

33. Multiply 365 by 2, by 3, and by 5. Add the three products and compare it with the multiplicand. Add the multipliers 2, 3, and 5.

34. Sound goes 1093 feet in a second; how far will it go in 5 seconds? The product is 185 feet more than a mile; how many feet in a mile?

35. The distance around a circle is about 3 times the distance across it. If it were *exactly* 3, how far would a bicyclist travel in riding 5 times around a circular track which is 352 feet across?

36. A bicyclist and a pedestrian start at the same time from Atlanta towards Chattanooga. The former travels 7 miles an hour, the latter 3; how far apart will they be in 1 hour? When they have traveled 16 hours? Draw diagram.

37. Washington was made president in 1789. In what year will the 27th presidential term end? A term is 4 years.

38. Iron weighs 8 times as much as water. If a certain amount of water weighs 41,543 pounds, how much would the same amount of iron weigh?

38. To multiply by 10, 100, 1000, etc.

Read Art. 22.

			Th.	H.	T.	U.
10 times 5 **U**nits	= 50 units	=			5	0
10 times 6 **T**ens	= 60 tens	=		6	0	
10 times 3 **H**undreds	= 30 hundreds	=	3	0		

We see from this that each order is multiplied by 10 by moving it one place to the left.

Th.	H.	T.	U.		Th.	H.	T.	U.
	3	6	5	$\times$ 10 =	3	6	5	0

$365 \times 10 \times 10 = 365 \times 100.$

$365 \times 10 \times 10 \times 10 = 365 \times 1000.$

To multiply by 10, 100, 1000, etc., annex as many naughts to the multiplicand as are in the multiplier.

EXAMPLES.

1. $365 \times 10.$	6. $365 \times 100.$	11. $365 \times 1000.$
2. $1728 \times 10.$	7. $1728 \times 100.$	12. $1728 \times 1000.$
3. $144 \times 10.$	8. $144 \times 100.$	13. $144 \times 1000.$
4. $62 \times 10.$	9. $62 \times 100.$	14. $62 \times 1000.$
5. $1492 \times 10.$	10. $1492 \times 100.$	15. $1492 \times 1000.$

16. A keg of nails weighs 100 pounds. What is the weight of 1728 kegs of nails?

17. There are 100 years in a century. How many times has the earth gone round the sun in 19 centuries?

18. A bicyclist and a pedestrian travel in opposite directions from Atlanta at the rate of 7 and 3 miles an hour respectively. How far apart will they be in 1 hour? After traveling 35 hours?

19. There are 100 cents in a dollar. How many cents in 6 dollars?

20. Silver is 10 times as heavy as water. A cubic foot of water weighs 62 pounds; what is the weight of a cubic foot of silver?

21. The great Chinese wall has a small tower every 100 yards. If a man walks beside it, how far has he gone after counting 251 spaces between the towers?

22. A boy starts on a bicycle from Atlanta, Ga., to Washington, D. C., a distance of 648 miles. He travels 63 miles a day; how far is he from Washington in 10 days?

23. Sound travels 1093 feet per second. If you see the puff of a steam-whistle 10 seconds before hearing it, how far are you from the whistle?

---

39. Copy these examples. Multiply first by 2, by 5, by 3, *then* annex the naught.

a.	**b.**	**c.**
473	831	231
20	500	3000
9460	415500	693000

EXAMPLES.

1. Multiply 473 by 2. Multiply the product by 10.
2. Multiply 831 by 5. Multiply the product by 100.
3. Multiply 231 by 3. Multiply the product by 1000.

4. 144×70.	7. 75×300.	10. 963×6000.
5. 1728×40.	8. 813×400.	11. 365×7000.
6. 212×90.	9. 483×500.	12. 492×8000.

13. A bushel of wheat weighs 60 pounds; how many pounds in 212 bushels?

14. A bushel of ear-corn (corn on the cob) weighs 70 pounds; how many pounds in 528 bushels?

15. A bushel of bran weighs 20 pounds; how many pounds in 75 sacks, each sack containing 2 bushels?

16. A degree of a great circle on the earth contains 60 geographical miles (also called *knots*). How many such miles in 360 degrees; that is, around the earth?

17. In a mile there are 5280 feet; how many feet in 80 miles?

18. The average length of human life is about 30 years. How many years are 197 such periods?

19. 67 people on the whole earth die each minute, how many die in an hour?

20. 70 babies on the whole earth are born each minute; how many are born in 24 hours?

21. How many minutes makes a day?

22. How many days in thirty years?

---

40. To multiply by any number.

Multiply 3617 by 245.

	3617	
	245	
5 times the multiplicand =	18085	Partial products.
40 times the multiplicand =	14468	
200 times the multiplicand =	7234	
245 times the multiplicand =	886165	

The right hand zeros of the partial products are omitted because they do not affect the result of addition. Notice and use as a rule, that *The right hand figure of each partial product is placed under the multiplier used.* 4 times 7 = 28; put **8** under 4. 2 times 7 = 14; put **4** under 2.

EXAMPLES.

31,416 multiplied by		Multiples.
1	=	31,416
2	=	62,832
3	=	94,248
4	=	125,664
5	=	157,080
6	=	188,496
7	=	219,912
8	=	251,328
9	=	282,744

a. How much is 31,416 × 5 ?

b. How much is 31,416 × 6 ?

c. How much is 31,416 × 3 ?

(See answers under multiples.)

d.

```
  31416
    365
 157080
188496
 94248
11466840
```

e.

```
  31416
    249
 282744
125664
 62832
7822584
```

Multiply 31,416 *by*

1. 41	**4.** 123	**7.** 649	**10.** 3412	**13.** 6819
2. 18	**5.** 235	**8.** 753	**11.** 3125	**14.** 4765
3. 63	**6.** 361	**9.** 869	**12.** 2354	**15.** 7198

62,622 multiplied by		Multiples.
1	=	62,522
2	=	125,244
3	=	187,866
4	=	250,488
5	=	313,110
6	=	375,732
7	=	438,354
8	=	500,976
9	=	563,598

Multiply 62,622 *by*

16. 41	**26.** 3412
17. 18	**27.** 3125
18. 63	**28.** 2354
19. 25	**29.** 6819
20. 36	**30.** 4765
21. 123	**31.** 7198
22. 235	**32.** 8543
23. 361	**33.** 6119
24. 649	**34.** 7984
25. 753	**35.** 8197

Multiply

365 *by*	1492 *by*	1607 *by*	6822 *by*
36. 23	46. 731	56. 863	66. 177
37. 36	47. 245	57. 491	67. 898
38. 49	48. 176	58. 365	68. 461
39. 18	49. 432	59. 745	69. 365
40. 75	50. 812	60. 913	70. 918
41. 27	51. 473	61. 492	71. 871
42. 68	52. 291	62. 784	72. 643
43. 92	53. 462	63. 314	73. 765
44. 87	54. 365	64. 159	74. 842
45. 95	55. 418	65. 943	75. 684

Multiply

76. 528 by 23.	84. 1492 by 731.	92. 7401 by 831.
77. 176 by 36.	85. 1607 by 245.	93. 1386 by 418.
78. 212 by 49.	86. 5280 by 176.	94. 4444 by 642.
79. 492 by 18.	87. 4162 by 432.	95. 6822 by 235.
80. 841 by 21.	88. 7842 by 812.	96. 3466 by 681.
81. 216 by 74.	89. 9143 by 473.	97. 4141 by 263.
82. 376 by 27.	90. 6418 by 291.	98. 2718 by 426.
83. 912 by 38.	91. 7302 by 462.	99. 3535 by 437.

41. Multiply 4183 by 2005.

```
 4183
 2005
20915
8366
8386915
```

The partial products by zeros are nothing, and are therefore omitted. *The first figure of the product by 2 must be placed under the 2.*

Examples.

Multiply

1. 1728 by 203.	3. 4761 by 407.	5. 7001 by 405.
2. 2173 by 107.	4. 4302 by 203.	6. 31,425 by 4001.

7. 31,415 by 3002.
8. 60,728 by 7005.
9. 45,302 by 5003.
10. 60,841 by 7004.
11. 415,846 by 30,003.
12. 613,052 by 40,004.
13. 714,085 by 50,505.
14. 323,233 by 30,103.
15. 430,215 by 50,404.

16. Georgia has 137 counties with an average area of 424 square miles in each county. What is the area of Georgia?

17. Find the total number of pounds in 35,004 bales of cotton if each bale weighs 503 pounds?

18. A barrel of flour weighs 196 pounds; how many pounds in 404 barrels?

19. Find the product of 4005 multiplied by itself?

20. A bicycle wheel rotates 705 times in going a mile; how often would it rotate in going from Atlanta to Augusta, Georgia, a distance of 171 miles?

21. The president receives $137 per day; how much is that per year (365 days)?

Multiply 5280 *by* 1600.

1st Step.

5280
1600
3168
528

DIRECTIONS.—Put the first integer of the multiplier under the first integer of the multiplicand, naughts falling on the right. Multiply by the integer and after adding partial products, annex the number of naughts in both multiplicand and multiplier.

2d Step.

5280
1600
3168
528
8448000

Multiply

22. 5280 by 2300.
23. by 250.
24. by 1760.
25. 31400 by 3700.
26. by 360.
27. by 1760.
28. 63300 by 1500.
29. by 480.
30. by 1760.

42. Numbers that are multiplied together are called *factors*. (Art. 80). $3 \times 5 = 15$; 3 and 5 are factors.

The dots right and left are called rows, up and down, columns.

In 1 row there are 5 dots; in 3 rows, 3×5 dots $= 15$ dots.

In 1 column there are 3 dots; in 5 columns; 5×3 dots $=$ 15 dots.

$$3 \times 5 = 5 \times 3.$$

43. *The product is the same in whatever order the factors are arranged.*

This is called the **commutative** law of multiplication.

$352 \times 3 \times 5 = 3 \times 5 \times 352$. $a \times b = b \times a$.

352		352
3	Notice that the two results are the same. On	15
1056	the left we multiply 352 by 3 and the product by	1760
5	5; on the right we multiply 352 by 15 (3×5).	352
5280		5280

Examples.

1. Multiply 728 by 2 and the product by 3. This is the same as multiplying by what number?
2. Multiply 365 by 15, using the factors of 15.
3. Multiply 1225 by 21, using factors.
4. Multiply 1492 by 28, using factors.
5. Multiply 5280 by 42, using factors.
6. Multiply 1895 by 35, using factors.
7. Multiply 149,605 by 36, using factors.
8. Multiply 365,192 by 21, using factors.
9. Multiply 405,405 by 24, using factors.
10. Multiply 340,043 by 56, using factors.

11. $22{,}584 \times 28$.	**16.** $186{,}504 \times 36$.	**21.** $31{,}416 \times 15$.
12. $31{,}525 \times 27$.	**17.** $186{,}504 \times 21$.	**22.** $31{,}416 \times 35$.
13. $22{,}584 \times 42$.	**18.** $186{,}504 \times 56$.	**23.** $31{,}416 \times 56$.
14. $31{,}525 \times 18$.	**19.** $186{,}504 \times 42$.	**24.** $31{,}416 \times 63$.
15. $22{,}584 \times 27$.	**20.** $186{,}504 \times 28$.	**25.** $31{,}416 \times 81$.

---

44. REVIEW.

1. What is multiplication ?

2. Name the parts in multiplication. $5x \times 3 = 15x$.

3. Define a concrete number; an abstract number.

4. Explain which parts in multiplication may be concrete and which abstract.

5. When the multiplier is 7 and the multiplicand $9x$, what is the product ?

6. What is the sign of multiplication ? Tell the two ways of reading this sign and the proper place of the multiplier.

7. The average daily attendance at the World's Fair was 149,605. The Fair was open 183 days; what was the total attendance ?

8. Light travels 186,500 miles in a second and it takes 493 seconds for the sun's light to reach the earth; how far off is the sun ?

9. You see a flash of lightning and in 14 seconds hear thunder. How far was the lightning, if sound travels 1093 feet per second ? (The answer is about three miles.)

10. What is the value of 1728 bales of cotton when the average weight of each bale is 473 pounds and cotton is worth 7 cents a pound ?

11. The area of the United States is 3,668,167 square miles. In 1890 there was an average 17 people to each square mile; what was the total population ?

Each member of the House of the National Congress in 1897 represented 173,901 people. Find the population of the following States:

		Number of Representatives.			Number of Representatives.
12.	New York,	34.	17.	Texas,	13.
13.	Pennsylvania,	28.	18.	Missouri,	15.
14.	Illinois,	22.	19.	Georgia,	11.
15.	Ohio,	21.	20.	Virginia,	10.
16.	Massachusetts,	13.	21.	California,	7.

22. On the whole earth there are 70 births and 67 deaths each minute. How much does the population increase in 24 hours?

DIVERSIONS.—23. What two numbers multiplied together produce seven?

24. What is the difference between six dozen dozen and half a dozen dozen?

25. What is the difference between twice twenty-five and twice five and twenty?

26. Multiply 10101 by 231.

Fill out the places marked x in the following with figures: (*x* stands for something unknown.)

27.	28.	29.	30.
$2x1$	365	$49x$	$8x2$
65	$x1$	$x5$	$4x$
1155	365	2460	$48x2$
1386	1460	2952	$32x8$
			36892

Multiply 84 by 76.

$$\begin{array}{r} 84 \\ 76 \\ \hline 5624 \\ 48 \\ 28 \\ \hline 6384 \end{array}$$

Notice whence the figures are derived and where the partial products are placed.

Multiply

31. 84 by 67.
32. 96 by 43.
33. 79 by 87.
34. 67 by 45.
35. 89 by 78.
36. 75 by 75.
37. 49 by 49.
38. 87 by 87.
39. 95 by 95.
40. 68 by 68.

In the following examples, what must have been the multiplier?

41. $\begin{array}{r} 1492. \\ x \\ \hline 5968 \\ 4476 \\ \hline \end{array}$ **42.** $\begin{array}{r} 1492. \\ x \\ \hline 7460 \\ 2984 \\ \hline \end{array}$ **43.** $\begin{array}{r} 365. \\ x \\ \hline 2555 \\ 1825 \\ \hline \end{array}$ **44.** $\begin{array}{r} 365. \\ x \\ \hline 3285 \\ 2920 \\ \hline \end{array}$

---

45. SUBTRACTION.

// // /. Here are 5 marks. How many must be added to make 8? // // / // /. As each new mark is made, say the whole number made, (six, seven, *eight*) till the required number are made; then count the number that have been added, (one, two, *three*). By this method you have learned that the difference between 8 and 5 is 3. This is the way to find the difference between two numbers when the difference and number subtracted are both digits. By remembering the differences for every pair of digits, there is a process called **subtraction** by which the difference between *any* two numbers may be found.

The sign of subtraction is — and is called **minus.**

$$8 - 5 = 3 \text{ is read 8 minus 5 equals 3.}$$

After careful consideration of the logic of both methods and experience in the school-room as to the results of both methods, the mak-

ing-up method for finding differences has been here adopted in preference to the taking-away method. One process is as logical as as the other; but the making-up method is more nearly related to addition and, therefore, to one who knows addition (as does the learner at this stage of his progress), it makes a less demand upon his conscious attention. The advantage of the method is perhaps most apparent in Art. 49, examples 8 to 12.

If the make-up method is not the superior, why, although taught differently, do shopkeepers almost universally use it in making change?

1. // // / and how many make 9? 5 + = 9?
2. // // and how many make 7? 4 + = 7?
3. // // // and how many make 11? 6 + = 11?
4. // // / and how many make 12? 5 + = 12?
5. // / and how many make 8? 3 + = 8?

How many added

6. to // // make 8?
7. to // // / make 7?
8. to // / make 9?
9. to // make 7?
10. to // // make 6?
11. to // // // make 11?
12. to // // // / make 13?
13. to // // / make 10?
14. to // // // / make 11?
15. to // // // make 14?

Point at the following examples and announce the differences:

The list contains the complete subtraction table, the digits being used either as subtrahends or differences.

	a.	b.	c.	d.
16.	11 — 6	5 — 3	14 — 9	10 — 7
17.	4 — 2	9 — 7	11 — 8	8 — 6
18.	13 — 7	7 — 4	12 — 6	18 — 9
19.	8 — 4	10 — 6	7 — 5	13 — 9
20.	17 — 9	14 — 8	13 — 8	12 — 7
21.	6 — 3	9 — 5	11 — 7	11 — 9

22.	8 — 5	14 — 7	9 — 6	12 — 8
23.	17 — 8	6 — 4	15 — 9	12 — 9
24.	9 — 7	10 — 5	10 — 8	16 — 8

In the following, speak only the differences; thus, 4, 14, 24, 34, 44, etc.

Subtract

25. 8 from 12, 22, 32, 42, 52, 62, 72.
26. 5 from 7, 17, 27, 37, 47, 57, 67.
27. 4 from 9, 19, 29, 39, 49, 59, 69.
28. 6 from 10, 20, 30, 40, 50, 60, 70.
29. 2 from 9, 4, 7, 12, 5, 8, 10, 6, 11.
30. 3 from 5, 8, 13, 6, 9, 11, 7, 12, 10.
31. 4 from 9, 14, 7, 10, 12, 8, 13, 11, 7.
32. 5 from 15, 8, 11, 13, 9, 14, 12, 8, 10.
33. 6 from 9, 12, 14, 10, 15, 13, 9, 11, 8.
34. 7 from 13, 15, 11, 16, 14, 10, 12, 9, 13.
35. 8 from 16, 12, 17, 15, 11, 13, 10, 14, 9.
36. 9 from 13, 18, 16, 12, 14, 11, 15, 10, 13.

45a. DEFINITIONS.

Subtraction is the process of finding how many units must be added to one number to make a given number.

The given number is called the **minuend.**

The number to be subtracted is called the **subtrahend.**

The result is called the **difference.**

8 — 5 = 3. 8 is the minuend; 5 is the subtrahend; and 3 is the difference.

a. Name the parts in 12 — 7 = 5; in 17 — 9 = 8.

b. Subtrahend = 5; minuend = 11; what is the difference?

c. Minuend = 15; subtrahend = 7; what is the difference?

Only *like* numbers can be subtracted.

46. A man having \$4769, spent \$1525. How much had he left?

\$4769
1525
\$3244

DIRECTIONS.—Write the numbers with the same orders under each other. Begin on the right and subtract.

1.	2.	3.	4.	5.
Dollars.	Bushels.	Pounds.	Miles.	Gallons.
763	8476	45,362	84,276	98,789
421	4371	42,132	43,241	44,332

6. From 896 dollars subtract 493 dollars.

7. From 7308 take 4105.

8. From Atlanta to San Francisco is 2,847 miles. After traveling from Atlanta 1,523 miles, how far are you from San Francisco?

9. The minuend is 85,613; the subtrahend 43,401; what is the difference?

10. 956,814 — 433,612 = ?

11. 986,487 — 743,255 = ?

12.	13.	14.	15.
864,942	376,518	684,296	743,891
332,741	255,313	284,173	621,531

47. A man having \$5281, spent \$1728; how much had he left?

	Th.	H.	T.	U.
\$	5	2	8	4
	1	7	2	8
\$	3	5	5	6

DIRECTIONS.—Write the numbers with the same orders under each other. Begin on the right to subtract. Say: 8 and 6 (write 6) are 14; 3 and 5 (write 5) are 8; 7 and 5 are 12; 2 and 3 are 5.

EXPLANATION.—We add and subtract by remembering the sums of every pair of digits. Now the sum of two digits can not be greater than 18, (9 + 9). Therefore, the digit which was added to 8 must

have been one which made the sum 14. In adding, we say 8 and 6 are 14, write 4 and carry 1; then we say 3 and 5 are 8. So, in subtracting we carry the 1 from 14 and say 3 and how many make 8; etc.

Dollars.	Bushels.	Pounds.	Miles.	
835	4372	7532	84,273	$71,543\,x$
472	2547	4265	56,647	$46,271\,x$

1. The distance through the earth at the poles is 7899 miles; at the equator, 7925 miles. What is the difference?

2. How many years has it been since Columbus discovered America in 1492? Since the declaration of independence in 1776? Since Washington was make president in 1789?

3. From New York to Liverpool is 3040 miles, and to San Francisco is 3269 miles; what is the difference?

Find the increase in population of the following States:

	1890.	1880.
4. Georgia,	1,837,353;	1,542,180.
5. Texas,	2,235,523;	1,591,749.
6. New York,	5,997,853;	5,087,871.
7. Pennsylvania,	5,258,014;	4,282,891.

8. The United States: in 1890, 62,622,250; in 1880, 50,155,783.

9. The area of the globe is 196,096,060 square miles, of which 143,621,181 square miles is water. How much land is there?

10. The population of England in 1891 was 27,482,104, and of Great Britain 37,888,153. What was the difference?

11. From 1,000,000 take 314,159.

12. In 1892 Georgia raised 1,100,000 bales of cotton; in 1893, 830,450 bales. What was the difference?

13. The average yearly production of wheat in the United States is 458,077,791 bushels. How much is that less than five hundred million?

14. In 1890 the number of children of school age in the United States was 18,799,864, and the number of actual pupils was 8,329,234. How many children were not attending school?

15. When the subtrahend is 62,622,250 and minuend a hundred million, what is the difference?

16. The careers of 12,483 young men were watched. In 10 years, whiskey had killed 634, smoking cigarettes killed 405, and 841 died of other troubles. How many were then living?

17. From 987,654,321 subtract 123,456,789.

18. To what number must 142,857 be added to make 285,714?

19. From 100,101,001 subtract 62,622,250.

20. What is the difference between the largest and smallest cotton crop mentioned on page 31?

For other examples see pages 31 and 32. Use the total population as minuend and number of either sex as subtrahend.

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48. REVIEW.

1. What is subtraction?

2. Name the parts in subtraction: $5 - 3 = 2$.

3. What is the sign of subtraction and its name?

4. When the minuend is 10, the subtrahend 7, what is the difference?

5. The distance around the earth is 24,899 miles; the distance through it is 7925. Subtract the latter from the

former and the successive remainders as many times as you can.

6. 23,423,423 — 9,056,273 = what ?

7. Use 5280 and successive remainders as minuends and 1760 as a subtrahend 3 times.

8. The attendance at the World's Fair in 1893 was 27,377,733; at the Paris Exposition in 1889, 32,354,111. What was the difference ?

9. What number must be added to 7,965,499 to make 13,642,809 ?

10. From the difference between 3285 and 456 subtract the difference between 19,011 and 17,455.

11. From the largest number that can be expressed by the figures 7, 2, 6, 3, and 5, take the smallest number that can be expressed by the same figures.

12. The sum of two numbers is 917; one is the difference between 693 and 1201; find the other.

---

49. ADDITION AND SUBTRACTION.

1. Three addends give the sum 864; two of the addends are 173 and 445, what is the other ?

2. The minuend is 243; the sum of 125 and the subtrahend is 271; what is the difference ?

3. The sum of the minuend and subtrahend is 763; the minuend is 527; what is the subtrahend and what the difference ?

4. Two separate examples in subtraction have equal subtrahends; the minuends are 643 and 428, and the sum of the differences is 823; what is the subtrahend of each?

5. The minuend is 22,222 greater than the subtrahend; the subtrahend equals the remainder; the remainder is 37,564; what is the minuend ?

6. The population of a city on January 1, 1897, was 94,743. During the year there were 2375 births, 1846 deaths, 947 people moved in, and 459 moved away. What was the population January 1, 1898?

7. Two boys rode on bicycles from Atlanta in opposite directions:

	◂—— Roy.			A.	Walter.		——▸
Days	3d.	2d	1st.	1st.	2d.	3d.	
Miles	63	65	71	68	73	59.	

At the end of the third day how far had each traveled, and how far were they apart?

Subtract the sum of 1492, 1776, and 365 from 5280.

5280	DIRECTIONS.—Place the numbers and draw the lines as shown. Beginning on the right, add by columns up to the upper line, then ask yourself how many more must be added to make the number above the line. In this example, say, adding first column: 5, 11, 13, and **7** make 20, (write 7 and carry 2). Adding second column: 8, 15, 24, and **4** make 28, (write 4 and carry 2). Adding third column: 5, 12, 16, and **6** make 22, (write 6 and carry 2). Adding fourth column: 3, 4, and **1** make 5, (write 1).
1492	
1776	
365	
1647	

In the following examples, add the numbers between the lines and subtract the sum from the number above the top line:

8.	9.	10.	11.	12.
12765	6004	39370	10,000	61,027
3872	1776	5760	31,416	16,093
5280	1492	365	31,416	16,093
1760	1898	3937	31,416	16,093

13. The crown used by the English sovereign contains 3,091 jewels. Of these, 1361 are brilliant diamonds; 1273 are rose diamonds, and 147 are table diamonds. How many other jewels are in the crown?

14. Length of rivers: Amazon, 3944; Nile, 3800; Mississippi, 2616. The sum of these three lengths and that of the Missouri, is 13,456. How long is the Missouri river?

---

50. DIVERSIONS.—Take a set of numbers which increase in value. Subtract each from the one next greater. The sum of the remainders and smallest number should equal the largest number.

1. Numbers.	Remainders.	2. Numbers.	3. Numbers.
275895	207465	864293	361420
68430	66287	215642	28419
2143	1947	38291	19142
196		6425	3562

Make up similar examples.

4. Take any number, say 365,847. Subtract 2 and prefix 2: 2,365,845; this will be the sum. Take any other number, placing it under the first one for addition. Under the second number write a third *so that the sum of the digits in the same orders of the 2d and 3d numbers shall be 9* (see example). Take any fourth number and write a fifth under it according to the directions in italics. The sum of the five numbers will be as stated.

365,847
258,241
741,758
429,580
570,419
2,365,847

Make up similar examples.

5.	6.	7.
314,159	149,265	17,761,607
261,348	810,990	52,801,444
614,280	946,000	15,651,583
2,314,157	2,149,263	217,761,605

51. DIVISION.

//////////////. Here are 15 marks. Separate them into groups of three marks each and count the number of groups. /// /// /// /// ///; 5 groups. This is the way to find a quotient (5) when it and the divisor (3) are both digits; but by remembering all such quotients, there is a process called **division** by which the quotient of *any* two numbers may be found.

The sign of division is ÷.

$15 \div 3 = 5$ is read 15 divided by 3 equals 5.

3 and 5 are how many? **Addition.** 3 and how many are 8? **Subtraction.** 3 times 5 are how many? **Multiplication.** 3 times how many are 15? **Division.**

52. *How many groups of*

1. // make 12 marks?
2. /// make 15 marks?
3. //// make 16 marks?
4. ///// make 15 marks?
5. ///// make 20 marks?
6. /// make 21 marks?
7. //// make 24 marks?
8. ///// make 30 marks?
9. /// make 30 marks?
10. //// make 28 marks?

The answer to each of these ten questions is called a **quotient.**

How many groups of //// make 14 marks?
Ans. //// //// //// //. Three groups and two marks.

How many groups of

11. /// make 13 marks?
12. /// make 17 marks?
13. //// make 22 marks?
14. //// make 14 marks?
15. ///// make 17 marks?
16. //// make 22 marks?
17. //// make 35 marks?
18. /// make 28 marks?
19. /// make 17 marks?
20. ///// make 34 marks?

The number of groups is called the quotient; the number of marks left is called the **remainder.**

21. 3 times what number is 15 ? $15 \div 3 =$
22. 3 times what number is 27 ? $27 \div 3 =$
23. 4 times what number is 20 ? $20 \div 4 =$
24. 4 times what number is 28 ? $28 \div 4 =$
25. 5 times what number is 30 ? $30 \div 5 =$
26. 6 times what number is 42 ? $42 \div 6 =$
27. 7 times what number is 63 ? $63 \div 7 =$
28. 8 times what number is 56 ? $56 \div 8 =$
29. 9 times what number is 72 ? $72 \div 9 =$
30. 9 times what number is 54 ? $54 \div 9 =$

53. *Point at the following examples and give the quotients:*

The list contains the complete division table, the digits being used either as divisors or as quotients. To give the quotient in 11 **a**, the pupil should ask himself, "6 times what number is 18?" and similarly for the others.

	a.	**b.**	**c.**	**d.**
1.	$18 \div 6$	$9 \div 3$	$36 \div 9$	$42 \div 7$
2.	$4 \div 2$	$81 \div 9$	$16 \div 8$	$36 \div 6$
3.	$35 \div 7$	$12 \div 4$	$24 \div 6$	$45 \div 9$
4.	$16 \div 4$	$12 \div 6$	$15 \div 5$	$72 \div 9$
5	$10 \div 5$	$40 \div 8$	$32 \div 8$	$49 \div 7$
6	$63 \div 9$	$25 \div 5$	$14 \div 7$	$54 \div 9$
7.	$6 \div 3$	$28 \div 7$	$30 \div 6$	$48 \div 8$
8.	$56 \div 8$	$8 \div 4$	$18 \div 9$	$27 \div 9$
9.	$21 \div 7$	$20 \div 5$	$24 \div 8$	$64 \div 8$

A number is not divided by 1. The operation may be expressed, but the result shows that no *division* has taken place.

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54. DEFINITIONS.

Division is a process for finding by what a number must be multiplied to produce a given number.

The parts in division are called **divisor, dividend, quotient,** and **remainder.**

$17 \div 3 = 5$ and remainder 2. Here 17 is the dividend, 3 the divisor, 5 the quotient, and 2 the remainder.

Name the parts in $21 \div 3 = 7$; in $63 \div 7 = 9$.

55. *Find what must have been the multiplier in the following examples:*

1.	2.	3.	4.	5.
231	432	233	412	408
693	864	699	1648	2448
6.	**7.**	**8.**	**9.**	**10.**
315	412	913	742	365
1575	2472	2739	5936	2555

56. SHORT DIVISION.

Divide 693 by 3. (See Art. 36.)

$$3\overline{)693}\quad 231$$

DIRECTIONS.—Draw a line about the dividend as shown. Divide each digit of the dividend by the divisor. By this process you have found that 3 must be multiplied by 231 to produce 693; or that 693 lines will make 231 groups of 3 lines each.

EXAMPLES.

1. $864 \div 2$.	6. $963 \div 3$.	11. $884 \div 4$.	16. $1505 \div 5$.
2. $428 \div 2$.	7. $663 \div 3$.	12. $848 \div 4$.	17. $1806 \div 6$.
3. $644 \div 2$.	8. $996 \div 3$.	13. $1284 \div 4$.	18. $2807 \div 7$.
4. $482 \div 2$.	9. $366 \div 3$.	14. $1648 \div 4$.	19. $1616 \div 8$.
5. $866 \div 2$.	10. $699 \div 3$.	15. $2084 \div 4$.	20. $2727 \div 9$.

57. Read Articles 23 and 28, II.

One unit of any order makes ten units of the next lower order. A figure of any order may be read as tens of the next lower order.

Divide 5184 by 3.

	1	7	2	8
	Th.	H.	T.	U.
3	5	1	8	4

DIRECTIONS.—Place the numbers and draw the line as shown. 3 is contained in 5 **once** and 2 over. This remainder 2 has not yet been divided: by reduction, it becomes 20 hundreds, and the 1 in hundreds place gives 21 hundreds; which divided by 3 gives **7**. Thus each remainder is simply *prefixed* to the next right hand order (which makes it *tens* of that order). 3 is contained in 8 **twice** and 2 over; and into 24, **8** times. *Copy this example.*

EXAMPLES.

Divide

1. 730 by 2.	**6.** 2555 by 7.	**11.** 9905 by 7.
2. 483 by 3.	**7.** 31,680 by 6.	**12.** 325,938 by 6.
3. 624 by 4.	**8.** 1696 by 8.	**13.** 69,055 by 7.
4. 852 by 4.	**9.** 2996 by 7.	**14.** 4347 by 9.
5. 786 by 3.	**10.** 4476 by 3.	**15.** 7460 by 5.

58. Divide 4378 by 4.

$$\begin{array}{r} 1094 \\ 4\overline{)4378}-2 \end{array}$$

When the divisor is not contained in the partial dividend it is so indicated by naught in the quotient. When there is a remainder, it should be written as is 2 in the example. This shows that if two were subtracted from the given number the exact quotient would be 1094.

1. 4376 ÷ 4.	**6.** 24,026 ÷ 6.	**11.** 37,869 ÷ 9.
2. 1826 ÷ 3.	**7.** 14,087 ÷ 7.	**12.** 26,404 ÷ 5.
3. 20,518 ÷ 5.	**8.** 8749 ÷ 8.	**13.** 9843 ÷ 9.
4. 12,411 ÷ 2.	**9.** 6343 ÷ 9.	**14.** 11,260 ÷ 7.
5. 46,808 ÷ 6.	**10.** 1221 ÷ 4.	**15.** 8749 ÷ 8.

16. Divide 5280 feet (one mile) by 3 feet. 3 feet make one yard; what is your quotient?

17. I heard the sound of a whistle 7 seconds after seeing the puff of steam, and, measuring the distance, found I was

7651 feet from the whistle. How far does sound go in 1 second?

18. Around a circle is 3 times across it. What is the distance, in feet, across a circle which is 1 mile (5280 feet) around?

19. How many steps of 4 feet each would a man take in running 5 miles? 1 mile = 5280 feet.

20. If a bicyclist goes 9 miles an hour, how many hours will he be in going from Boston to Atlanta, a distance of 1089 miles?

21. A bicycle wheel is 7 feet around. How often will it rotate in going 1 mile (5280 feet)?

22. Iron weighs 8 times as much as water. A certain quantity of iron weighs 8744 pounds; what does the same quantity of water weigh?

23. In 7 square miles there are 4480 acres; how many acres in 1 square mile?

24. 9 cubic feet of water weigh 738 pounds; what does 1 cubic foot of water weigh?

25. How many 5 grain capsules can be filled with 5760 grains (1 pound) of quinine?

26. A boy said: If I had 3 times my number of marbles and 5 more, I should have 185. How many have I?

27. A dog is 225 feet behind a rabbit. The rabbit runs 16 and the dog 21 feet a second. How many seconds before the dog will catch the hare?

28. A boy bought papers at 4 for 8 cents and sold them at 5 cents apiece. How many papers had he sold when his gain was 525 cents?

29. 9 cubic feet of water weigh 558 pounds; what is the weight of 1 cubic foot?

30. If a bicyclist rides 455 miles in 7 days, how far does he ride in a day? in 5 days?

31. If the Mississippi flows 3 miles per hour, how many hours does it take for water from its source, Lake Itasca, to reach the Gulf of Mexico? The Mississippi is 3162 miles long.

32. Mr. King paid $9500 for a farm, and sold it for $8000, losing $3 per acre; how many acres were in the farm?

59. The sign of division is ÷; but division may also be indicated by writing the divisor under the dividend with a line between.

$$12 \div 3 = \frac{12}{3} = 12 \text{ divided by } 3.$$

In this way indicate the following divisions:

	a.	b.	c.	d.
1.	$12 \div 3$.	$24 \div 7$.	$4 \div 9$.	$17 \div 83$.
2.	$15 \div 5$.	$19 \div 11$.	$7 \div 13$.	$14 \div 97$.
3.	$18 \div 2$.	$27 \div 15$.	$1 \div 3$.	$73 \div 141$.
4.	$17 \div 6$.	$35 \div 8$.	$2 \div 5$.	$62 \div 187$.
5.	$14 \div 3$.	$18 \div 13$.	$14 \div 23$.	$49 \div 301$.

60 Division consists in dividing the dividend and then each successive remainder by the divisor. The last remainder is always smaller than the divisor and therefore cannot be actually divided, but the division should always be *indicated* as above.

$\frac{14}{3} = 4\frac{2}{3}$. $\frac{19}{11} = 1\frac{8}{11}$. $\frac{1549}{23} = 67\frac{8}{23}$.

Remainders written in this way are called **fractions.** (Art. 106.) The figure *above* the line is called the **numerator;** the figure *below* the line is called the **denominator.** When the denominator is small, a fraction is read by reading its numerator as a cardinal number and its denominator as an ordinal number.

Cardinal numbers are one, two, three, four, etc. Ordinal numbers are, first, second, third, fourth, etc.

$\frac{2}{3}$ is read two-thirds. $\frac{7}{9}$ is read seven-ninths.

Read:

$\frac{2}{3}$ $\frac{8}{11}$ $\frac{9}{14}$ $\frac{5}{8}$ $\frac{4}{9}$ $\frac{5}{19}$

$\frac{7}{9}$ $\frac{4}{7}$ $\frac{3}{4}$ $\frac{7}{12}$ $\frac{8}{23}$ $\frac{3}{5}$

$\frac{1}{2}$ is read one-*half* instead or one-second.

61. When the denominator is large, a fraction is usually read as an expression of division. $\frac{27}{145}$ is read, 27 divided by 145.

Read:

$\frac{27}{145}$ $\frac{18}{273}$ $\frac{45}{761}$ $\frac{123}{877}$ $\frac{25}{1492}$

$\frac{14}{3} = 4\frac{2}{3}$ is read, 14 thirds equals 4 and $\frac{2}{3}$.

$\frac{2328}{761} = 3\frac{45}{761}$. The quotient is read, 3 and 45 divided by 761.

Write the quotients to the following:

1. $\frac{23}{3}$.
2. $\frac{17}{4}$.
3. $\frac{29}{6}$.
4. $\frac{125}{7}$.
5. $\frac{435}{9}$.
6. $\frac{67}{2}$.
7. $\frac{432}{7}$.
8. $\frac{685}{3}$.
9. $\frac{1492}{5}$.
10. $\frac{1607}{4}$.
11. $\frac{1093}{6}$.
12. $\frac{5280}{7}$.
13. $\frac{1728}{7}$.
14. $\frac{5760}{7}$.
15. $\frac{1776}{9}$.

---

62. LONG DIVISION.

Divide 14,880 by 32.

```
  465
32|14880
  128
  ---
  208
  192
  ---
   160
   160
   ---
```

DIRECTIONS.—Place the numbers and draw the lines as shown. Find each figure of the quotient as in short division, using only the left figure of the divisor. Thus, 3 is contained in 14, 4 times; multiply the whole divisor, 32, by this figure of the quotient, subtract, and bring down the next figure, 8. 3 is contained in 20, 6 times; multiply 32 by 6, subtract, draw down next figure, and so on.

Place each digit of the quotient over the right hand digit of the dividend used in obtaining it.

Copy this example and study the directions.

EXPLANATION.

$$\begin{array}{rcl} 32 \times 400 &=& 12800 \\ 32 \times 60 &=& 1920 \\ 32 \times 5 &=& 160 \\ \hline 32 \times 465 &=& 14880 \end{array}$$

Obtain from this the products $\left\{ \begin{array}{l} 32 \times 65, \\ 32 \times 460, \\ 32 \times 405. \end{array} \right.$

32 *multiplied by*

2 = 64. 6 = 192.
3 = 96. 7 = 224.
4 = 128. 8 = 256.
5 = 160. 9 = 288.

Divide by 32:

1. 800. 6. 11,680.
2. 1024. 7. 13,600.
3. 1312. 8. 13,504.
4. 2336. 9. 17,744.
5. 2684. 10. 31,296.

Divide

11. 14,880 by 32.
12. 11,680 by 32.
13. 800 by 32.
14. 2336 by 32.
15. 13,600 by 32.
16. 9963 by 41.
17. 40,768 by 52.
18. 27,531 by 63.
19. 29,575 by 91.
20. 11,985 by 51.
21. 18,615 by 51.
22. 42,903 by 63.
23. 27,405 by 63.
24. 14,175 by 75.
25. 39,600 by 75.
26. 26,062 by 83.
27. 20,632 by 72.
28. 22,608 by 72.
29. 66,584 by 82.
30. 28,290 by 82.
31. 24,722 by 94.
32. 35,814 by 94.
33. 62,220 by 85.
34. 20,825 by 85.

63. Divide 1541 by 23.

$$\begin{array}{r} 67 \\ 23\overline{)1541} \\ 138 \\ \hline 161 \\ 161 \\ \hline \end{array}$$

DIRECTIONS.—Follow directions in Art. 62. Using the left figure of the divisor as before, we say, 2 is contained in 15, 7 times; but multiplying 23 by 7 gives 161, which is more than 154. Therefore 23 is not contained in 154 as many as 7 times. Try the next lower figure, 6, and so on.

Copy this example till you can perform the division perfectly.

From this example, you see that the left figure of the divisor is simply used as a *trial* divisor. There is no other way to find the true figure of the quotient than to make these trials.

EXAMPLES.

1. 1426 ÷ 23.	11. 109,935 ÷ 35.	21. 14,322 ÷ 231.
2. 736 ÷ 23.	12. 9855 ÷ 27.	22. 7200 ÷ 225.
3. 1288 ÷ 23.	13. 85,248 ÷ 48.	23. 24,192 ÷ 432.
4. 5313 ÷ 23.	14. 96,980 ÷ 65.	24. 53,361 ÷ 231.
5. 3312 ÷ 23.	15. 17,808 ÷ 84.	25. 133,125 ÷ 365.
6. 8642 ÷ 58.	16. 18,212 ÷ 58.	26. 165,792 ÷ 528.
7. 7807 ÷ 37.	17. 30,624 ÷ 58.	27. 92,928 ÷ 176.
8. 4847 ÷ 37.	18. 21,170 ÷ 58.	28. 133,225 ÷ 365.
9. 5513 ÷ 37.	19. 10,256 ÷ 58.	29. 435,600 ÷ 528.
10. 6549 ÷ 37.	20. 41,412 ÷ 58.	30. 118,096 ÷ 176.

64. When the divisor is not contained in the partial dividend, it is so indicated by naught in the quotient. Indicate the remainders as explained in Art. 60.

Execute these divisions:

1. $365\overline{)399072}$ $= 1093\frac{1}{3}\frac{2}{6}\frac{7}{5}$

2. $144\overline{)760417}$ $= 5280\frac{9}{144}\frac{7}{}$

3. 231,747 ÷ 212.	8. 2,102,423 ÷ 365.
4. 451,401 ÷ 144.	9. 371,300 ÷ 231.
5. 334,800 ÷ 314.	10. 335,000 ÷ 1066.
6. 231,819 ÷ 1093.	11. 451,391 ÷ 3141.
7. 371,943 ÷ 1607.	12. 2,102,573 ÷ 5760.

65. TO DIVIDE BY 10, 100, ETC.

To multiply a number by 10, each digit is moved one place to the *left*, (Art. 38); conversely, to divide by 10, each digit should be moved one place to the *right*.

Remove the digits of 1492 one place to the right.

This is done by putting a period between 9 and 2 to show that the integral part of the number ends with 9.

Th.	H.	T.	U.			Th.	H.	T.	U.
1	4	9	2	÷ 10 =		1	4	9	.2

To divide by 100, or 10 × 10, point off two places.

To divide by 1000, or 10 × 10 × 10, point off three places, etc.

$$1492 \div 100 = 14.92. \qquad 1492 \div 1000 = 1.492.$$

$$\frac{1492}{100} = 14.92 = 14\tfrac{92}{100}$$

The integral (Art. 77 d.) quotient is the number to the left of the period and the number on the right is the remainder.

$$\begin{array}{r|l} & 14\frac{92}{100} \\ 100 & 1492 \\ & 100 \\ \hline & 492 \\ & 400 \\ \hline & 92 \end{array}$$

1. 1492 ÷ 100.	6. 492 ÷ 10.	11. 1092 ÷ 1000.
2. 1728 ÷ 100.	7. 145 ÷ 10.	12. 5280 ÷ 1000.
3. 1092 ÷ 100.	8. 365 ÷ 10.	13. 3141 ÷ 1000.
4. 5280 ÷ 100.	9. 843 ÷ 10.	14. 6721 ÷ 1000.
5. 3141 ÷ 100.	10. 921 ÷ 10.	15. 4873 ÷ 1000.

66. Since 500 = 5 × 100; 170 = 17 × 10; etc., we have

RULE: *To divide by a number ending in naughts, point off the naughts in the divisor and the same number of places in the dividend before dividing.*

$$\begin{array}{r|l} & 211\frac{649}{2500} \\ 25.00 & 5281.49 \end{array}$$

The remainder obtained by using the divisor without naughts should be *prefixed* to the figure cut off from the dividend. The reason is apparent from the example on the right.

$$\begin{array}{r|l} & 211\frac{649}{2500} \\ 25.00 & 5281.49 \\ & 50 \\ \hline & 28 \\ & 25 \\ \hline & 31 \\ & 25 \\ \hline & 649 \end{array}$$

1. 6248 ÷ 40.	11. 31,685 ÷ 600.	21. 314,159 ÷ 3000.
2. 7869 ÷ 30.	12. 32,593 ÷ 500.	22. 784,291 ÷ 6000.
3. 2555 ÷ 70	13. 69,055 ÷ 200.	23. 477,125 ÷ 2500
4. 4347 ÷ 90	14. 48,347 ÷ 800.	24. 648,192 ÷ 3200.
5. 7462 ÷ 50.	15. 25,004 ÷ 400.	25. 829,146 ÷ 2100.
6. 3146 ÷ 20.	16. 31,416 ÷ 500.	26. 271,828 ÷ 2300.
7. 5280 ÷ 30.	17. 86,191 ÷ 700.	27. 314,159 ÷ 4500.
8. 3937 ÷ 20.	18. 24,222 ÷ 300.	28. 141,421 ÷ 1600.
9. 7462 ÷ 50.	19. 36,524 ÷ 600.	29. 154,323 ÷ 3700.
10. 3146 ÷ 60.	20. 39,371 ÷ 900.	30. 223,606 ÷ 5900.

67. AVERAGES.

365	The **average** of several numbers	609
416	is a number which will produce the	609
984	same sum if put in the place of each	609
671	of the given numbers. It is found	609
2436	by dividing the sum by the number	2436
	of numbers. The average of the	
	four numbers on the left is 609.	

$$2436 \div 4 = 609.$$

What is the average of

1. 5, 6, and 9.
2. 2, 6, and 7.
3. 3, 4, and 8.
4. 6, 7, 8, and 11.
5. 5, 7, 10, and 14.
6. 7, 9, 11, and 13.

7. 5 bales of cotton weigh respectively 480, 503, 491, 475, and 511 pounds. What is the average weight?

8. Several bushels of wheat weighed respectively 61, 59, 58, 60, 59, and 63 pounds. What was the average weight?

9. The daily attendance at Ivy Street School for a week was 425, 432, 429, 437, and 432. What was the *average* daily attendance ?

10. 6 careful experiments were made to find the velocity of sound; the results in feet per second were 1093, 1096, 1104, 1092, 1087, and 1090. What was the average ?

11. John and Henry are each 15 years old; Mary is 14, and Roy and Sam each 13. What is the average age of these five ?

12. In a certain class two of the pupils are 10 years old; six are 11; twelve are 12; and ten are 13. What is the average age of all these pupils ?

13. What is the average of 6, 7, 8, 9, and 10 ? What is the average of 6 and 10 ?

14. What is the average of 5, 7, 9, 11, and 13 ? What is the average of 5 and 13 ?

What is the average of the following:

15. 4, 7, 10, 13, and 16. Of 4 and 16 ?
16. 1, 5, 9, 13, and 17. Of 1 and 17 ?
17. 2, 7, 12, 17, and 22. Of 2 and 22 ?
18. 5, 11, 17, 22, and 27. Of 5 and 27 ?

} Notice the difference.

19. Count by 2's from 3 to 15. Find the sum of these numbers, including 3 and 15.

20. Count by 3's from 5 to 23. Find the sum of these numbers, including 5 and 23.

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68. REVIEW.

1. What is division ?

2. Name the parts in 17 divided by 3 equals 5 and 2 over.

3. Explain which parts may be concrete and which abstract numbers.

4. Define Short and Long divisions.

5. What is the trial divisor?

6. Show the two ways of indicating division.

7. What is an average?

8. Which parts are the same in multiplication and division? Div. $15x \div 3x = 5$. Mul. $3x \times 5 = 15x$.

9. If the divisor and quotient are given, how do you find the dividend?

10. If the dividend and quotient are given, how do you find the divisor?

11. If the divisor, quotient, and remainder are given, how do you find the dividend?

12. The hot and cold water faucets of a bath tub which holds 128 gallons are turned on. 21 gallons of hot water flow in 3 minutes and 63 gallons of cold water in 7 minutes. How long before the tub is half full?

13. A ship travels 23 miles per hour. How many hours will it take to go from New York to Liverpool, 3040 miles?

14. A train averages 25 miles in going from New York to San Francisco, 3269 miles. How many hours does it take to make the trip?

15. The area of Georgia is 58,000 square miles and the population in 1890 was 1,837,353. What was the average number of people per square mile?

16. New York. Area 47,000 square miles; population in 1890, 5,997,853. Average population per square mile?

17. The President of the United States receives $50,000 in 365 days (1 year). How much is that per day?

18. It is 25,000 miles around the earth. How many miles per day must one travel to go round in 80 days?

19. The World's Fair was open 183 days; the total attendance was 27,377,733. What was the average per day?

20. A typewriter writes 25 pages in 45 minutes. Each page has 25 lines with an average of 10 words to the line. How many words can she write per minute ?

21. There are 16 ounces in a pound. How many pounds in 4625 ounces ?

22. How many square miles in Georgia ? There are 37,120,000 acres, and 640 acres make one square mile.

23. In 1893 Georgia raised 1,732,824 bushels of wheat on 15,200 acres. What was the average yield per acre ?

24. In 1893 Georgia raised 33,678,000 bushels of corn on 149,680 acres. What was the average yield per acre ?

25. The earth turns on its axes 360 degrees in 24 hours. How many degrees does it turn in 1 hour ?

26. If gold is worth $240 per pound, how many pounds of gold are in one million dollars ?

27. What is the nearest number to 11,575 that will contain 221 without a remainder ?

28. How many bushels of corn will weigh as much as 224 bushels of wheat ? A bushel of wheat weighs 60 pounds; of corn 56 pounds.

29. If 16 gold dollars weigh as much as one silver dollar, how many silver dollars weigh as much as 2480 gold dollars?

30. A cubic foot of gold weighs 1203 pounds, and is worth $362,380; what is one pound of gold worth ?

31. In the Civil War the entire Union force was 2,653,062 men. If 1 for every 65 was killed, and 1 for every 10 wounded, how many were killed and how many were wounded ?

32. 10 eggs weigh one pound. How many pounds will 15 dozen eggs weigh ?

33. DIVERSION.—A piece of cloth is 50 yards long. If you cut off two yards every day, how many days will it take to cut it into two-yard pieces ?

34. 231 cubic inches make a gallon. How many gallons in 2,333,331 cubic inches?

35. A certain number is multiplied by 95 and 225 is subtracted from the product; the remainder divided by 125 gives a quotient 37. What is the number?

SUGGESTION.— $125 \overline{)\text{Remainder}}^{37}$ Remainder $+ 225 =$ product.

It requires 173,901 people for each representative in the House of Representatives of our national Congress. To how many representatives is each of the following States entitled and how many people are left unrepresented?

		Population.			Population.
36.	New York,	5,997,853.	41.	Texas,	2,235,523.
37.	Pennsylvania,	5,258,014.	42.	Missouri,	2,679,184.
38.	Illinois,	3,826,351.	43.	Georgia,	1,837,353.
39.	Ohio,	3,672,316.	44.	Virginia,	1,655,980.
40.	Massachusetts,	2,238,943.	45.	California,	1,208,130.

---

69. SIGNS OF AGGREGATION.

Parentheses ( ), are used to enclose numbers upon which *one* operation is to be performed. $7 + (\ )$ means that the quantity in parentheses is to be added to 7.

$7 - (\ )$, the quantity is to be subtracted from 7.
$(\ ) \times 7$, the quantity is to be multiplied by 7.
$(\ ) \div 7$, the quantity is to be divided by 7.

The **vinculum** $\overline{\quad}$, **brackets** [ ], and **braces** $\left\{\ \right\}$ are used like parentheses.

They are called sign of aggregation.

$$7 + (8 - 3) = 7 + \overline{8 - 3} = 7 + [8 - 3] = 7 + \left\{ 8 - 3 \right\}$$

RULE.—*Find the value of the expression in parentheses before performing the indicated operation.*

$7 + (8 - 3)$. Find the value of $8 - 3$ before adding.
$8 - 3 = 5$; $7 + 5 = 12$.

$7 - \overline{8 - 3} = 7 - 5 = 2$. $[8 - 3] \times 7 = 5 \times 7 = 35$.

Multiplying and dividing should be performed *before* adding or subtracting.

1. $9 + (11 - 4)$.
2. $9 - (11 - 4)$.
3. $(11 - 4) \times 9$.
4. $(67 - 4) \div 9$.
5. $(7 + 4) - (11 - 5)$.
6. $\overline{3 + 4} - 2 \times 3$.
7. $\overline{4 + 3} \times 2 - \overline{11 - 3} \div 4$.
8. $(25 - 3 \times 5 + \overline{11 - 4} \times 2) \div 4$.
9. $56 \div 7 + 12 \times 3 - 52 \div 2$.
10. $(491 - \overline{275 - 95}) \times (45 - 16)$.
11. $81 - \overline{(108 \div 9) + 43}$.
12. $148 - \overline{15 + 31} \div 6$.
13. $4 \times (15 \times 7 \div 35) + \overline{17 - 2} \times 18 \div 5$.
14. $10 \times 9 \div 15 - (\overline{4 + 3} - \overline{7 - 2}) \times 2$.

When multiplication and division signs occur they should be taken in order from the *left*.

---

70. REVIEW OF FUNDAMENTAL OPERATIONS.

1. *Name the results of each fundamental operation.*
2. What kind of numbers can be added or subtracted?
3. Make the signs to indicate each operation.
4. Make the signs for equality, dollars, and aggregation

5. A student studies or recites 6 hours per day during 5 days of each week for 4 college sessions of 36 weeks each. How many hours does he work during the entire course?

6. A and B bought 284 acres of land. A's share was 24 acres more than B's. What was the share of each?

7. The sum of two numbers is 500. If 125 be subtracted from their sum, the result will be 75 more than the larger number. What is the smaller number?

8. Of what number is 365 both the divisor and quotient?

9. Out of 9,642,000 boys in the United States, if cigarettes kill 1 and injure 15 in every 100, how many remain uninjured from this cause?

10. A pleasure party, consisting of 12 women and 8 men, remained out 14 days. The expenses averaged a dollar a day each. The men shared equally the whole bill. What was each man's share?

11. A school boy is 10 minutes late every day. How many minutes does he lose in a session of 180 days? At 60 minutes to the hour and 5 hours to the day, to how many school days is this equal?

12 When a train is moving 50 miles an hour, how many telegraph poles will it pass per minute? 1 mile = 1760 yards. Telegraph poles are placed 66 yards apart. 1 hour = 60 minutes.

13. A man drove to town at the rate of 8 miles per hour and after remaining awhile, returned at the rate of 6 miles. He was traveling just 6 hours. How far was it to town?

14. If 26 be used as an addend 39 times, how much less than 1225 will the sum be?

15. Simplify $(30 \div \overline{2 + 3} - 4) \times 9 + \overline{8 - 3} \times 6$.

16. A rabbit is 45 yards ahead of a dog, but can run only 25 yards while the dog runs 28 yards; how many yards must the dog run to catch the rabbit?

71. SOME PROPERTIES OF NUMBERS.

The remainder obtained after dividing a number by 9 is the same as that obtained after dividing the sum of its digits by 9.

417 ÷ 9 gives a remainder 3. The sum of 4, 1, and 7 is 12, and 12 ÷ 9 also gives a remainder 3. But the remainder of 12 ÷ 9 is found by adding the digits of 12. Hence the remainder of *any* number divided by 9 may be found by adding the digits of the number; if this sum be greater than 9, add *its* digits, and so on.

Number.	Sum of digits		
	of number	of 48	of 12
7,388,976	**48**	**12**	**3**

and 3 is the remainder after dividing 7,388,976 by 9.

72. The reason for the statement in italics is as follows:
Dividing 10, 100, 1000, etc., by 9 gives always a remainder 1.
Dividing 20, 200, 2000, etc., by 9 gives always a remainder 2.
Dividing 30, 300, 3000, etc., by 9 gives always a remainder 3.
Dividing 40, 400, 4000, etc., by 9 gives always a remainder 4, etc.

[Try some of these divisions.]

$$\begin{aligned}
3000 \div 9 &= 333 \text{ 9's} + \text{a remainder } 3. \\
100 \div 9 &= 11 \text{ 9's} + \text{a remainder } 1. \\
20 \div 9 &= 2 \text{ 9's} + \text{a remainder } 2. \\
1 \div 9 &= 0 \text{ 9's} + \text{a remainder } 1. \\
\hline
3121 \div 9 &= 346 \text{ 9's} + \text{a remainder } 7.
\end{aligned}$$

But 7 is the sum of the digits of 3121.

73. What will be the remainder when each of the following numbers is divided by 9?

1. 1492.	5. 24,899.	9. 78,631.	13. 1664.
2. 5280.	6. 31,416.	10. 42,590.	14. 1733.
3. 1776.	7. 43,560.	11. 78,133.	15. 2541.
4. 3141.	8. 84,263.	12. 65,482.	16. 6479.

74. This property of the number 9 gives an excellent method for proving the fundamental operations and of special practical use as applied to multiplication. It is called **casting out nines.**

ADDITION.

			Add.
1776 =	197	9's +	3.
5280 =	586	9's +	6.
1607 =	178	9's +	5.
3142 =	349	9's +	1.
11805 =	1310	9's +	15.
=	1311	9's +	6.

DIRECTIONS.—Divide each addend by 9, writing only the remainders. Add the remainders and, if larger than 9, divide it by 9. The remainder thus obtained should be the same as that obtained when the sum to be proved is divided by 9. It is unnecessary to find or write the numbers 197, 586, etc.

This method of proof could be employed by using other numbers as well as 9; 9 is used because of the ease with which remainders may be found when a number is divided by 9; and it is the remainders only that are needed in the proof.

Using the numbers in Art. 73, find sums and prove them by casting out nines.

Use examples in addition and prove the sums by casting out nines.

Subtraction.

	Subtract.
87,162	6
61,753	4
25,409	2

The example will explain itself.

Use examples under subtraction.

Multiplication.

	Multiply	
54743	5	
273	3	
164229	15	6
383201		
109486		
14944839		6

Use examples under multiplication.

Division. 5

```
  3     653
345|225553
    2070
    ----
     1855
     1725
     ----
      1303
      1035
      ----
       268
```

$345 \times 653 + 268 = 225{,}553.$

Cast out nines from each number.

$$\begin{array}{rcl} 3 \times 5 + 7 & & 4 \\ 15 + 7 & & \\ 22 & & \\ 4 & = & 4 \end{array}$$

7

RULE.—Cast out 9's from divisor, quotient, and remainder. Multiply together the numbers thus obtained from the divisor and quotient, add the number obtained from the remainder, and cast out the 9's from this sum. The number obtained should be the same as that obtained from casting out the 9's from the dividend.

75. Where the result of addition, subtraction, etc., is too great or too small by an exact number of 9's, this method evidently fails to *prove*; for casting out the 9's gives still the same remainder as if the answer were correct. But the probability of making such an error is so small that great confidence may be placed in the method.

In the following examples tell by the method of casting out 9's whether the answers are correct or not.

	1.	2.	3.	4.	5.
	1728	449	7284	684	3845
	365	876	9782	365	309
	5280	984	8447	492	1849
Supposed sum	7473	2319	25513	1541	6103

	6.	7.	8.	9.	10.
	835	4372	7532	84,273	71,543
	472	2547	4265	56,647	46,271
Supposed difference	463	1725	3267	28,626	25,272

		Supposed product.			Supposed quotient.	Remainder.
11.	3617 × 245	886,165.	16.	1541 ÷ 23	53	
12.	528 × 23	12,154.	17.	9855 ÷ 27	365	
13.	9143 × 473	4,323,629.	18.	53,367 ÷ 231	231	6
14.	6418 × 291	1,867,638.	19.	7200 ÷ 225	32	
15.	7401 × 831	54,036.	20.	24,192 ÷ 432	44	11

Casting out 9's to prove multiplication is a very practical operation and should be much used. It is not of so much importance as applied to the other operations.

76. DIVERSIONS.— 1. Think of a number. Subtract from it the sum of its digits: strike out any digit from the remainder; tell me the sum of the remaining digits and I will tell you the digit struck out.

DIRECTIONS.—Subtract the sum of the digits of the number told you from 9: the remainder will be the digit struck out.

2. Think of any number; divide it by 9 and tell me the remainder. Strike out any digit from the number thought of, divide by 9, tell me the remainder and I will tell you the digit struck out.

DIRECTIONS.—If the second remainder is less than the first, the digit struck out is the difference of the remainders; but if the second remainder is the greater, add 9 to the *first* remainder and then take their difference.

3. Think of a number composed of two digits not the same. Transpose the digits: take the difference between this number and the one thought of: name one of the digits and I will name the other.

DIRECTIONS.—Subtract the one he names from 9.

4. Think of any number. Divide it by 9 and tell me the remainder. Multiply the number thought of by (here I name a multiplier). Divide the product by 9 and the remainder will be (here I name the remainder).

DIRECTIONS.—Multiply the first remainder by the multiplier; divide the product by 9 and the remainder will be the one required.

5. Think of a number greater than 3. Multiply it by 3: if the product is even divide by 2; if odd, add 1 and then divide by 2. Multiply the quotient by 3: if this product is even, divide by 2; if odd, add 1 and then divide by 2. Now divide by 9, tell me the quotient (without the remainder) and I will tell you the number thought of.

DIRECTIONS.—If even both times multiply the quotient by 4. If odd both times, multiply by 4 and add 3. If even first and odd second multiply by 4 and add 2. If odd first and even second, multiply by 4 and add 1.

77.

A number is either:

Odd or Even	Prime or Composite	Integral or Fractional	Concrete or Abstract	Cardinal or Ordinal.

a. An **odd number** is one not divisible by 2; as 9, 45.

b. An **even number** is one divisible by 2; as 8, 76.

c. A **prime number** is one divisible only by itself; as, 7, 29.

All prime numbers except 2 are odd.

Two numbers are prime to each other when no number will divide them both. Thus, 8 and 15, though not prime numbers, are prime to each other.

d. An **integral number,** or an *integer*, is a whole number; as 9, 22.

e. A **fractional number** is one containing a fraction; as, $\frac{2}{3}$, $5\frac{1}{2}$, 7.4

---

78. TESTS OF DIVISIBILITY.

A number is divisible—

By **2** when its last digit is even or naught. **18, 130.**

By **4** when the number expressed by the last two digits is divisible by 4. 10312, 1724.

By **8** when the number expressed by the last three digits is divisible by 8. 71824, 13016.

By **3** when the sum of its digits is divisible by 3. 645 is divisible by 3 because $6 + 4 + 5 = 15$ is divisible by 3.

By **9** when the sum of its digits is divisible by 9. $7 + 3 + 8 + 9 = 27$. Therefore any number composed of the digits 7, 3, 8, and 9 is divisible by 9; as, 8379, 9873.

By **5** when the last digit is 5 or 0. 835, 710.

Since a number whose last digit is 0 is divisible by both 2 and 5 it is also divisible by $2 \times 5 = 10$.

By **6** when it is divisible by both 2 and 3; that is, when the last digit is naught or even and the sum of the digits is divisible by 3.

By **7** by trial only.

By **11** when the difference between the sums of the digits in the even and odd places is 0 or divisible by 11. 782,056 is divisible by 11 because $6 + 0 + 8 = 14$, $5 + 2 + 7 = 14$, and $14 - 14 = 0$.

---

79. EXAMPLES.

Tell by inspection what numbers will divide

	a.	b.	c.	d.
1.	57.	927.	1793.	1515.
2.	38.	279.	326.	1818.
3.	85.	87.	1803.	5555.
4.	138.	604.	2931.	16,824.
5.	428.	5608.	1772.	46,266.
6.	7288.	3421.	8271.	77,110.

FACTORS.

80. A **factor** is one of two or more integers which are used to make a product.

$2 \times 3 \times 5 = 30$. Either 2, 3, or 5 is a factor of 30.

A factor will always exactly divide the product and give an integral quotient.

What are the factors of 15? of 21? of 6? of 14?

a. A number that has factors besides itself and 1 is called **composite** and is a *multiple* of each factor.

b. A power is the product when the same factor is repeated. $5 \times 5 \times 5 = 125$; 125 is a power of 5.

The factor itself is the first power. When the factor is used twice, the product is the second power. When the factor is used three times, the product is the third power. When the factor is used four times, the product is the fourth power, etc.

c. Instead of 5×5, we may write 5^2; 5 to the second power. Instead of $5 \times 5 \times 5$, we may write 5^3; 5 to the third power. Instead of $5 \times 5 \times 5 \times 5$, we may write 5^4; 5 to the fourth power, etc.

d. The small figure placed above and to the right to show how many times the factor is to be repeated is called an **exponent.**

---

81. Prime Factors of a Number.

To resolve a number into its prime factors is to find what prime numbers multiplied together produce it.

What are the prime factors of 3080?

$$\begin{array}{r|l} 2 & 3080 \\ 2 & 1540 \\ 2 & 770 \\ 5 & 385 \\ 7 & 77 \\ & 11 \end{array}$$

DIRECTIONS.—Divide the given number by any prime number which, by inspection, you know will divide it. Continue thus to divide the successive quotients till the the quotient itself is a prime number. The divisors and last quotient will be the prime factors sought, because they will all be prime numbers and when multipled together will produce the given number.

$$3080 = 2 \times 2 \times 2 \times 5 \times 7 \times 11,$$
$$= 2^3 \times 5 \times 7 \times 11.$$

The rules in Art. 78 are given to assist in telling when a number is divisible by 2, 3, 5, and 11. There is no prime number larger than 11 in the following examples:

What are the prime factors of

1. 42.	7. 462.	13. 726.	19. 1617.
2. 30.	8. 1925.	14. 9075.	20. 1815.
3. 105.	9. 1575.	15. 5929.	21. 3630.
4. 88.	10. 2156.	16. 13,475.	22. 1980.
5. 180.	11. 600.	17. 11,025.	23. 1260.
6. 1155.	12. 198.	18. 2750.	24. 7350.

For other examples employ the numbers used as terms of the fractions in Art. 123.

82. If any prime number larger than 11 is a factor it must be found by trial. The number of trials, however, may be limited by this rule: Divide by prime numbers in their order of increase till you find a factor, or *till the quotient is less than the divisor.* In the latter case the number itself is prime. For example, has the number 431 factors or is it prime? Dividing 431 in succession by 2, 3, 5, 13, 17, and 19, we find none of them to be factors and in every case

the quotient is greater than the divisor. [Perform these divisions.] Dividing by 23, the quotient is 18 with a remainder; hence 431 is a prime number. For, if it were composite, both divisor and quotient would be factors. But by trial we found that it had no factors less than 23 and since the quotients become less as the divisors become greater than 23, there is no use to try further.

83. *The following list contains the first thirty prime numbers.*

1	13	37	61	89
2	17	41	67	97
3	19	43	71	101
5	23	47	73	103
7	29	53	79	107
11	31	59	83	109

84. *Are the following numbers prime or composite?*

1. 323.	**4.** 503.	**7.** 1067.	**10.** 463.
2. 853.	**5.** 881.	**8.** 601.	**11.** 283.
3. 756.	**6.** 997.	**9.** 3267.	**12.** 209.

Find the prime factors of the following numbers:

13. 11,011.	**17.** 882.	**21.** 870.	**25.** 15,925.
14. 9350.	**18.** 1067.	**22.** 1843.	**26.** 9350.
15. 13,981.	**19.** 1173.	**23.** 1921.	**27.** 15,675.
16. 582.	**20.** 841.	**24.** 8085.	**28.** 1115.

This process of resolving a number into its prime factors is used in finding, by one method, the greatest common divisor and least common multiple of numbers; to be explained later.

Greatest Common Divisor.

85. A common divisor of two or more numbers is an exact divisor of each number.

3 is a common divisor of 18 and 27 because it will exactly divide each number.

The Greatest Common Divisor (G. C. D.) of two or more numbers is just what the words imply—their *greatest* common divisor.

9 is the greatest common divisor of 18 and 27 because it is the greatest number that will exactly divide them both.

86. We know by inspection that $3 \cdot 5$ divided by 3 gives 5 and divided by 5 gives 3.

A dot placed between numbers means that they are to be multiplied together. $5 \cdot 3 = 5 \times 3$.

It is evident, also, that $2 \cdot 3 \cdot 5 = 6 \cdot 5 = 10 \cdot 3 = 2 \cdot 15 = 30$. Hence, 30 is divisible by 2, 3, 5, 6, 10 and 15.

A number is divisible by each of its prime factors and the product of any two or more of them and by nothing else. [Commit this to memory.]

$70 = 2 \cdot 5 \cdot 7$ is divisible by 2, 5, 7, 10, 14 and 35.

Every number is divisible by itself.

What numbers will exactly divide

1. $2 \cdot 3 \cdot 5$.
2. $3 \cdot 7 \cdot 11$.
3. $2 \cdot 5 \cdot 7$.
4. $5 \cdot 7 \cdot 7$.
5. $3 \cdot 5 \cdot 7$.
6. $3 \cdot 5 \cdot 5$.
7. $2 \cdot 3 \cdot 5 \cdot 7$.
8. $3 \cdot 5 \cdot 7 \cdot 11$.
9. $2 \cdot 3 \cdot 3 \cdot 5$.

$105 = 3 \cdot 5 \cdot 7$.

$165 = 3 \cdot 5 \cdot 11$.

105 is divisible by 3, 5, 7, 15, 21, and 35.

165 is divisible by 3, 5, 11, 15, 33, and 55.

3, 5, and **15** are the only common divisors, therefore, **15** is the *greatest* common divisor.

87. *The G. C. D. of two or more numbers is the product of all the prime factors common to the numbers.*

$30 = 2 \cdot 3 \cdot 5$.
$105 = 3 \cdot 5 \cdot 7$.
$330 = 2 \cdot 3 \cdot 5 \cdot 11$.

3 and 5 will divide them all; hence 3×5 is their G. C. D.

What is the G. C. D. of

$5 \cdot 7$.	$2 \cdot 3 \cdot 5$.	$2 \cdot 3 \cdot 5 \cdot 7$.
1. $5 \cdot 11$.	4. $2 \cdot 3 \cdot 7$.	7. $2 \cdot 3 \cdot 5 \cdot 11$.
$5 \cdot 13$.	$2 \cdot 3 \cdot 11$.	$2 \cdot 3 \cdot 5 \cdot 5$.
		$2 \cdot 3 \cdot 5 \cdot 3$.
$3 \cdot 7$.	$2 \cdot 5 \cdot 7$.	$5 \cdot 7 \cdot 11 \cdot 13$.
2. $7 \cdot 11$.	5. $3 \cdot 5 \cdot 7$.	8. $2 \cdot 5 \cdot 7 \cdot 11$.
$5 \cdot 7$.	$5 \cdot 5 \cdot 7$.	$3 \cdot 5 \cdot 7 \cdot 11$.
		$5 \cdot 7 \cdot 7 \cdot 11$.
$11 \cdot 13$.	$5 \cdot 7 \cdot 7$.	$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$.
3. $2 \cdot 11$.	6. $7 \cdot 7 \cdot 7$.	9. $2 \cdot 2 \cdot 3 \cdot 3$.
$11 \cdot 11$.	$7 \cdot 7 \cdot 11$.	$2 \cdot 2 \cdot 3 \cdot 3 \cdot 13$.
		$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$.

See Art. 80, **d.**

$3 \cdot 7^2 \cdot 11$.	$2^3 \cdot 5^2 \cdot 7$.
10. $7^2 \cdot 11 \cdot 13$.	11. $2^3 \cdot 5^2 \cdot 11$.
$5 \cdot 7^2 \cdot 11$.	$2^3 \cdot 5^2 \cdot 13$.

12. $2^2 \cdot 5^2$.
$2^5 \cdot 5^3$.

13. $2^3 \cdot 7^3$.
$2 \cdot 7^2$.

14. $2^3 \cdot 5^3 \cdot 7$.
$2^3 \cdot 5^2 \cdot 7$.
$2^3 \cdot 5^2 \cdot 7$.

15. $3^2 \cdot 5 \cdot 7$.
$3^2 \cdot 5 \cdot 7 \cdot 11$.
$3^2 \cdot 5 \cdot 7 \cdot 7$.
$3^2 \cdot 5 \cdot 13$.

16. $2^3 \cdot 3^2 \cdot 5 \cdot 7$.
$2^3 \cdot 3^2 \cdot 7$.
$2^3 \cdot 3^2 \cdot 13$.
$2^3 \cdot 3^2 \cdot 5 \cdot 5$.

17. $5^3 \cdot 7^2 \cdot 11$.
$5^2 \cdot 7^3 \cdot 13$.
$5^4 \cdot 7^2$.
$5^2 \cdot 7^4$.

18. $3^2 \cdot 5^3 \cdot 11^4$.
$2^3 \cdot 3^2 \cdot 5^3 \cdot 11^2$.
$2^4 \cdot 5^2 \cdot 7 \cdot 11$.
$3^3 \cdot 5^3 \cdot 7^2 \cdot 11$.

19. $2^2 \cdot 3^2 \cdot 5^3 \cdot 7^3 \cdot 11$.
$2^2 \cdot 3^2 \cdot 5^3 \cdot 7^3$.
$3^2 \cdot 5^2 \cdot 7^3 \cdot 11^3$.
$2^2 \cdot 3^2 \cdot 5^3 \cdot 7^3 \cdot 13$.

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88. Second Method for Finding G. C. D.

When it is not easy to resolve the numbers into their prime factors, this method is preferable.

14 = 7 twos.
8 = 4 twos.
6 = 3 twos.

Since 14 and 8 are both divisible by 2, their difference, 6, is also divisible by 2.

A common divisor of two numbers is also a divisor of their difference.

Since a common divisor of two numbers must divide their difference, no common divisor can be greater than their difference. The G. C. D. of 15 and 18 is their difference, 3.

What is the G. C. D. of

1. 15 and 18.
2. 12 and 16.
3. 15 and 20.
4. 21 and 28.
5. 56 and 63.
6. 48 and 56.
7. 45 and 54.
8. 33 and 44.
9. 26 and 39.
10. 48 and 60.
11. 63 and 72.
12. 49 and 56.
13. 63 and 70.
14. 84 and 91.
15. 88 and 96.

89. What is the G. C. D. of 21 and 35 ?

Their difference is 14, but 14 will not divide either number. Now, whatever will divide both 21 and 35 will also divide 14; it will, therefore, divide the difference between 14 and either of these numbers. Take the difference between 14 and the smaller number because it will be the smaller difference and still be as large as the greatest common divisor could be.

21—14 = 7, and we find by trial that 7 will divide both 21 and 35.

Find the G. C. D. of 45 and 63.

45	63
18	45
27	18
18	
9 G. C. D.	

Subtract 45 from 63, leaving 18. Try 18. Subtract 18 from 45, leaving 27; subtract 18 from 27, leaving 9. Try 9. It is the G. C. D. When the remainder is larger than the subtrahend, continue to subtract the subtrahend.

What is the G. C. D. of

1. 21 and 35 ?
2. 24 and 40 ?
3. 27 and 45 ?
4. 33 and 55 ?
5. 39 and 65 ?
6. 45 and 75 ?
7. 45 and 81 ?
8. 45 and 63 ?
9. 35 and 49 ?
10. 28 and 36 ?
11. 63 and 81 ?
12. 63 and 91 ?
13. 104 and 136 ?
14. 169 and 221 ?
15. 49 and 119 ?

90. Division is a short method of subtraction; therefore, when many subtractions are to be performed, it is better to do them by division.

DIRECTIONS.—*Divide the larger number by the smaller, then the divisor by the remainder continously, until there is no remainder. The last divisor is the G. C. D.*

What is the G. C. D. of 133 and 190.

```
          1
133|190
    133      2
     57|133
        114     3
G. C. D.  19|57
            57
```

Copy this work and note whence the numbers come. The arrangement on the right is to be preferred. The quotients are of no consequence when the work is finished.

```
   133 | 190
   114 | 133
    19 |  57
G. C. D. |  57
```

91. Students will have no difficulty in *finding* the G. C. D. They may omit this explanation and learn it when more advanced.

EXPLANATION.—Since 9 is a divisor of 45 = 5 nines,
and of 27 = 3 nines,
it is also a divisor of 45 + 27 = 8 nines,
and of 45 — 27 = 2 nines.

Principle I.—*A divisor of two numbers is also a divisor of their sum and of their difference.*

9 will divide 18, it will divide 18 + 18 (Prin. I.), or any number of eighteens. Hence,

Principle II.—*A divisor of any number is also a divisor of any multiple* (Art. 95.) *of that number.*

Now, apply these principles to the example above.

The G. C. D. of 133 and 190, *whatever it is*, will divide their difference (I.). It will also divide 2 times 57 (II.).

Since it will divide 133 and 114, it will divide their difference, 19 (I.). Hence, it cannot be greater than 19. But 19 will divide 57; therefore, it will divide 2 times 57 (II.), and $114 + 19 = 133$ (I.), and also $133 + 57 = 190$ (I.).

Hence, since the G. C. D. of 133 and 190 cannot be greater than 19 and 19 *will* divide them, it is their G. C. D.

---

92. GEOMETRICAL EXPLANATION.

The problem is, find the longest distance which, when applied to A and B, will be contained exactly in each of them; that is, find the G. C. D. of A and B. Now, whatever will measure A and B will measure that part of B which is just as long as A, and, consequently, the remainder of B. But whatever will measure the remainder, R, cannot be greater than R; hence the G. C. D. of A and B cannot be greater than R. But will R divide them? If it will divide the smaller line A, it will also divide the larger line B; for, if it divides A, it will divide that part of B which is just as long as A and, of course, itself; that is, A + R, which = B.

Applying R to A, it is contained in it twice. Therefore, R is also contained in B and is the G. C. D. of A and B.

EXAMPLES.

What is the G. C. D. of

1. 187 and 323?
2. 133 and 209?
3. 700 and 756?
4. 882 and 1575?
5. 582 and 1067?
6. 841 and 870?

7. 1073 and 4181? 10. 1843 and 1921? 13. 990 and 3267?
8. 113 and 791? 11. 323 and 1615? 14. 8085 and 15,925?
9. 890 and 1869? 12. 420 and 1115? 15. 9350 and 15,675?

For other examples see Art. 100.

93. To find the G. C. D. of *several* numbers, find the G. C. D. of any two of them, then of that result and a third, and so on till all the numbers have been used. The last G. C. D. will be the one sought. For, each G. C. D. will contain all the factors common to the numbers that have been used, and the last G. C. D. will contain all the factors common to all the numbers.

What is the G. C. D. of 285, 190, and 133 ?

The G. C. D. of 285 and 190 is 95; of 95 and 133 is 19; therefore, the G. C. D. of 285, 190, and 133 is 19.

EXAMPLES.

What is the G. C. D. of

1. 4977, 3555, and 6636 ?
2. 1827, 3906, and 4599 ?
3. 1273, 6231, and 4087 ?
4. 791, 1243, and 1469 ?
5. 3850, 7700, and 10,780 ?
6. 6327, 5985, and 5130 ?
7. 4578, 5986, and 6867 ?
8. 407, 814, and 444 ?
9. 915, 1098, and 1830 ?
10. 730, 438, and 886 ?

---

94. PRACTICAL APPLICATIONS.

1. What is the greatest equal distance apart that posts can be placed around a triangular-shaped field whose sides are 840, 408, and 760 feet ? Draw a triangle.

2. A man owns three farms containing respectively 270, 594, and 945 acres. He wishes to divide them into equal

sized lots, each lot to contain as many acres as possible. How many acres will each lot contain ?

3. A courtyard which measures 1197 inches on two sides and 1463 inches on the other two, is to be paved with square tiles as large as possible. What will be the size of each tile?

4. What is the greatest number that will divide 223 and 244 and give the remainder 13 from each ?

5. A man has on deposit $625, $875, and $1125 in three different banks. What is the largest sum he can draw, if he draws on his deposits for equal amounts each time until the deposits are exhausted ?

The G. C. D. of numbers is used chiefly in reducing fractions to their lowest terms. See Art. 123.

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95. Least Common Multiple.

A multiple is the result obtained by multiplying.

This is also called a product, but the word multiple is used with reference to *one* factor only, whereas, a product is used with reference to all the factors.

a. A multiple of a number is exactly divisible by the number.

12 is a multiple of 3 because it is divisible by 3.

Name some multiple of

5, 3, 2, 4, 7, 8, 9, 11, 6, 12.

Of what number is 6 a multiple ? Of what number is

9 ? 10 ? 12 ? 15 ? 18 ? 21 ? 24 ?

Is 17 a multiple of 5 ? Is 24 a multiple of 8 ? Is 57 a multiple of 9 ?

b. A common multiple of two or more numbers is a number which is divisible by each given number. 24 is a common multiple of 3 and 4 because it is divisible by each of them.

Give some common multiple of

2 and 3.	4 and 6.	3 and 4.	9 and 12.
2 and 5.	8 and 6.	5 and 7.	10 and 15.

c. The Least Common Multiple (L. C. M.) of two or more numbers is the least number divisible by each of them.

The multiples

Of 6 are 6, 12, 18, 24, 30, **36,** 42, 48, 54, 60, 66, 72, etc.
Of 9 are 9, 18, 27, **36,** 45, 54, 63, 72, etc.
Of 12 are 12, 24, **36,** 48, 60, 72, etc.

36 is the L. C. M. of 6, 9, and 12, because it is the least number divisible by each of them.

The number whose factors are 2, 3, and 5 is divisible by each of these factors and by their product taken every way possible. Therefore, $2 \cdot 3 \cdot 5$, is the L. C. M. of 2, 3, 5, 6, 10, and 15.

$3 \cdot 5 \cdot 7$ is the L. C. M. of 15, 21, and 35.

Of what is

1. $3 \cdot 5$ the L. C. M.?
2. $2 \cdot 5 \cdot 7$ the L. C. M.?
3. $5 \cdot 7 \cdot 11$ the L. C. M.?
4. $3 \cdot 7 \cdot 11$ the L. C. M.?
5. $2 \cdot 5 \cdot 11$ the L. C. M.?
6. $3 \cdot 5 \cdot 7$ the L. C. M.?
7. $3 \cdot 5 \cdot 11$ the L. C. M.?
8. $2 \cdot 7 \cdot 11$ the L. C. M.?
9. $2 \cdot 3 \cdot 7$ the L. C. M.?
10. $5 \cdot 7 \cdot 11$ the L. C. M.?

96. To Find the L. C. M. of Several Numbers.

Method I. *Resolve each number into its prime factors and select from these all the factors, and no more, necessary to produce each number. The product of these factors will be the L. C. M. sought.*

What is the L. C. M. of 42, 66, and 105?

$42 = 2 \cdot 3 \cdot 7.$
$66 = 2 \cdot 3 \cdot 11.$
$105 = 3 \cdot 5 \cdot 7.$
A. $2 \cdot 3 \cdot 7.$
B. $2 \cdot 3 \cdot 7 \cdot 11.$
L. C. M. $2 \cdot 3 \cdot 7 \cdot 11 \cdot 5.$

The factors necessary to produce 42 are 2, 3, and 7; set these aside, A. To produce 66 we must have, besides factors already in A, the factor 11; and to produce 105, the additional factor 5. Therefore, $2 \cdot 3 \cdot 7 \cdot 11 \cdot 5$ is divisible by 42, 66, and 105.

Art. (repeated) 86. *A number is divisible by each of its prime factors and the product of any two or more of them and by nothing else.*

A L. C. M. must, therefore, contain all the prime factors (and no others) found in each of the given numbers.

The L. C. M. of $2 \cdot 3$ and $2 \cdot 7$ must be divisible by 2, 3, and 7 and, therefore, must contain these factors.

97. *What is the L. C. M. of* [Give factors only of answer.]

$3 \cdot 5.$	$2 \cdot 3 \cdot 11.$	$2^3 \cdot 3 \cdot 7.$
1. $3 \cdot 7.$	**6.** $3 \cdot 5 \cdot 7.$	**11.** $2^3 \cdot 5 \cdot 11.$
$5 \cdot 7.$	$5 \cdot 7 \cdot 11.$	$2^3 \cdot 5 \cdot 7.$
$2 \cdot 13.$	$2 \cdot 2 \cdot 13.$	$2^3 \cdot 3^2 \cdot 7.$
2. $2 \cdot 5.$	**7.** $2 \cdot 3 \cdot 7.$	**12.** $3^2 \cdot 5^2 \cdot 11.$
$3 \cdot 5.$	$3 \cdot 7 \cdot 11.$	$5^2 \cdot 7 \cdot 11.$
$7 \cdot 7.$	$3 \cdot 3 \cdot 5.$	$5^2 \cdot 7^4 \cdot 11.$
3. $5 \cdot 5.$	**8.** $5 \cdot 5 \cdot 7.$	**13.** $5^3 \cdot 11^3 \cdot 13.$
$5 \cdot 7.$	$3 \cdot 5 \cdot 7.$	$11 \cdot 13^4 \cdot 17.$
$2 \cdot 5.$	$3 \cdot 5 \cdot 5.$	$2^3 \cdot 5^2 \cdot 11.$
4. $3 \cdot 5.$	**9.** $7 \cdot 7 \cdot 11.$	**14.** $3^2 \cdot 7 \cdot 11.$
$5 \cdot 5.$	$5 \cdot 5 \cdot 7.$	$2^3 \cdot 5 \cdot 7.$
$3 \cdot 5.$	$3 \cdot 13 \cdot 7.$	$5^2 \cdot 7 \cdot 11.$
5. $2 \cdot 7.$	**10.** $5 \cdot 7 \cdot 13.$	**15.** $7^3 \cdot 11^2 \cdot 13.$
$5 \cdot 7.$	$5 \cdot 5 \cdot 7.$	$5 \cdot 7^3 \cdot 13^2.$

98. By inspecting the example solved in Art. 96, it may be seen that a factor found in two or more of the given numbers is used but once in the L. C. M. For instance, 2 is found in both 42 and 66, but is used only once in the L. C. M. From this fact we derive

Method II. *Place the numbers in a horizontal row and divide by any prime number that will divide two or more. Bring down in the row of quotients those numbers that cannot be divided. Continue this process till no two numbers have a common factor. The product of all the divisors and the last quotients is the L. C. M. of the given numbers.*

2	42	66	105
3	21	33	105
7	7	11	35
		11	5

L. C. M. $= 2 \times 3 \times 7 \times 11 \times 5$.

99. *What is the L. C. M. of*

1. 30, 105, and 175?
2. 78, 105, and 110?
3. 66, 105, and 385?
4. 156, 330, and 385?
5. 300, 630, and 735?
6. 4, 10, 15, and 25?
7. 11, 35, 49, and 77?
8. 1800, 2700, and 4500?
9. 15, 34, 102, and 137?
10. 273, 330, 495, and 546?
11. 16, 40, 96, and 105?
12. 1, 2, 3, 4, 5, 6, 7, 8, and 9?
13. 10, 12, 15, and 75?
14. 45, 75, 135, and 180?
15. 16, 60, 140, and 192?
16. 48, 54, 112, and 312?
17. 36, 42, 48, and 126?
18. 63, 72, 84, and 105?
19. 14, 35, 45, and 140?
20. 39, 77, 231, and 429?

100. APPLICATION.

1. What is the width of the narrowest walk that can be paved with blocks 45 inches long and 30 inches wide, the blocks being placed either across or with the length of the walk.

2. What is the shortest piece of calico that can be cut into pieces 6, 10, or 15 yards long and nothing remain ?

3. A, B and C ride at a constant rate on a circular bicycle track. A rides round once in 15 minutes, B in 18 minutes, and C in 20 minutes. If they start together, how long before they will be together again ? How many rounds will each have made ?

4. How many acres in the smallest farm that can be exactly divided into lots of 25, 30, 35, or 40 acres each ?

5. The front wheel of a bicycle is 9 feet and the rear wheel 6 feet in circumference. A marked spoke in each wheel is vertical at the same instant. How far will the bicycle go before the spokes are again both vertical ?

6. How far will the bicycle have gone if the spokes have both been vertical 293 times ? (See Example 5.)

101. DIVERSIONS.—1. A wishes to expend the smallest equal sums possible in the purchase of sheep at \$12, cows at \$20, and horses at \$75 or \$90 a head. Which priced horses should he buy ? What sum will he expend, and how many of each will he buy ?

2. An old woman, on being asked how many eggs she had, could only remember that in counting them into the basket by 2's, 3's, 4's, or 5's there always remained one over. What is the smallest number she could have had ?

102. REVIEW.

1. $1894 \times 365 = 691{,}310$. Prove by casting out 9's.
2. When does casting out 9's fail to prove?
3. What is a prime number? An integer?
4. Give the tests of divisibility for 2, 3, 4, 5, 6, 9, and 11.
5. What is a factor? A power? An exponent?
6. Resolve 13,475 into its prime factors.
7. Is the number 18,473 prime or composite?
8. By what numbers is a given number divisible? Art. 64.
9. Find by second method the G. C. D. of 990 and 3267.
10. What is a multiple?
11. Give one method for finding the L. C. M. of several numbers.
12. Find the L. C. M. of 1729 and 2093. Give reason for process.
13. How do you find the G. C. D. of three or more numbers? Art. 93. The L. C. M? Art. 101.
14. Compare the product of 1540 and 3003 with the product of their L. C. M. and G. C. D.
15. Take two numbers: 11×7 and 11×13.

Their G. C. D. is 11. Their L. C. M. is $11 \times 7 \times 13$.

The product of the numbers is $11 \times 7 \cdot 11 \times 13$.

The product of their G. C. D. and L. C. M. is $11 \cdot 11 \times 7 \times 13$.

It will be noticed that the two products contain the same factors and, from the nature of the work, that the product of any two numbers is equal to the product of their G. C. D. and L. C. M.

The product of two numbers is $13^2 \times 5 \times 11$. Their L. C. M. is $13 \times 5 \times 11$; what is their G. C. D.? One of the numbers is 13×5, what is the other?

16. The product of the G. C. D. and L. C. M. of two numbers is 2695: one of the numbers is 35, what is the other? Find their G. C. D. and L. C. M.
17. The L. C. M. of two numbers is 6930; their G. C. D. is 45: one of the two numbers is 630, what is the other?

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